On the Determination of the Stable Bed Slope of a Channel Using Mathematical Model

František Křovák

Department of Land Use and Improvement, Faculty of Forestry and Environment, Czech University of Life Sciences in Prague, Prague, Czech Republic

Abstract: The paper deals with an analysis of selected equations used for the determination of a stable longwise slope calculation of torrential rivers. Irregularity of the gradient, accompanied by heavy bed-load experiencing abrupt changes of the flow as a result of heavy rainfalls of short duration and high intensity, these are typical features impacting the behaviour and characteristics of torrential rivers. The determination of the stable bottom slope, when the river bed is kept unpaved but still provides resistence against harmfull effects of rapids, becomes an essential objective of the study. Three methods are used to determine the stable slope: the first is based on tangent tension (shear stress theory), the second observes a (critical) non-scouring cross-sectional velocity (critical mean channel velocities), and the third applies the bottom layer velocity, (the critical bed velocities). The mathematical hydraulic model HEC-RAS v. 3.1.3 has been used for the verification of the methods in the Jindřichovický creek case study in the Krušné hory.

Keywords: stable bed slope; shear stress theory; critical mean channel velocities; critical bed velocities; HEC-RAS model

Typically, the hydraulics of torrential rivers is quite different if compared with those of lowlands. Irregularities of the lengthwise river-bed gradient and a significantly varying grain-size distribution of the bed-load are specific features of such rivers (Novák 1981).

The bed-load is usually blended and it consists of sandy, gravelly, and cobble-formed grain particles. Boulders can also appear in some of the mountainous creeks (Novák 1988).

Sudden changes in the flow rate triggered by flash rainfall of short duration and high intensity usually hit solely small drainage areas; this is also a typical feature of such channels behaviour (Křovák & Kovář 2005).

High flow rate results in losses of the bed-load from the channel bottom and from the river banks. Thus, the sediment deposition during the decrease of the driving force becomes an unavoidable consequence. One of the basic objectives of the respective studies is the determination of the stable bed slope of the channel that would resist the driving force during the design floods. The creation of a sustainable bottom slope depends not only on the sediment grain-size distribution, but also on the saturation with water of the bed-load. The process is quite complex and not yet formulated in a comprehensive manner (Dvořák & Novák 1994; Gordon et al. 1996).

The theoretical scope of the study aims mainly at three methods of the stable bed slope analysis. The methods are based on the shear stress theory, on the critical mean channel velocities distribution, and on the critical bed velocity that is based on the bottom velocities.

The hydraulic model HEC-RAS v. 3.1.3 has been used for the method verification in the Jindřichovický creek in Krušné hory.

MATERIAL AND METHODS

The choice of the methods listed below is not accidental, only those have been applied where adequate data were available. With respect to the restricted extent of the paper, the complete formulation of the governing equations and their theoretical reasoning were omitted. The reader is therefore referred to the respective references, e.g. Nᴏᴠᴀ́ᴋ (1988) or Lᴏᴘᴇᴢ (1993). Similarly, the methods of the standard (effective) grain size determination are excluded from further considerations.

(1) Methods based on critical shear stress (after Shields)

$$i_s = \frac{0.06 \times (\rho_m - \rho) \times d_e}{\rho \times R} \tag{1}$$

(2) Methods based on critical average channel velocities (Manning-Strickler)

$$i_s = \frac{v_v^{\,2}}{k_s^2 \times R^{4/3}} \tag{2}$$

(3) Methods based on critical bed velocities (Nᴏᴠᴀ́ᴋ 1988)

$$i_s = 0.0035 \times C^2 \times \frac{d_e}{R} \tag{3}$$

The following equation has been used for the conversion of *k* and *n* coefficients,

$$n = \frac{R^{1/6}}{18 \times \log_{10} \times (12.2 \times \dfrac{R}{k}\,)} \tag{4}$$

where:
i_s – stable bed slope (m/m)
ρ_m – bed-load material density (kg/m^3)
ρ – water density (kg/m^3)
d_e – effective grain diameter (m)
R – hydraulic radius (m)
v_v – critical mean channel velocities (m/s)
k – coefficient, bottom roughness (m)
n – Manning's roughness coefficient
k_s – mean velocity coefficient of wetted perimeter, unpaved channel bed (after Strickler)

a – the constant in Stickler's equation related to the value of d_e (Dᴠᴏʀ̌ᴀ́ᴋ & Nᴏᴠᴀ́ᴋ 1994)
C – characteristic of the sediment load (after Nᴏᴠᴀ́ᴋ 1988)

The hydraulic model HEC-RAS has been used to quantitatively analyse the above equations.

Structure of the model

The mathematical hydraulic model HEC-RAS (Hydrologic Engineering Center's River Analysis System) came into consideration as an optimal tool for the solution of the sediment transport within the river studied. The program HEC-RAS uses an integrated environment of MS Windows and provides an outstanding graphical user's interface (GUI) in the detailed hydraulic analysis of the open channel flow and of the flow control measures in both artificial and natural channels. It works both in steady and unsteady conditions. The calculation requires the setting up of three principal data categories: the river-bed and control-structures geometry, the hydraulic losses, and the boundary conditions. It is with advantage to use the links to CAD and GIS systems also in 3D display already built into the modelling system. The channel capacity can be investigated by two principle means of solutions for hydraulic open channels analyses, flow control structures included:

(1) To solve the passage of the design flood wave by hydraulic model based on numerical solution of unsteady flow process. Such technique requires hydrographs input at upper boundary profile of the reach and, similarly to the further, the detailed description of geometric and hydraulic river-bed parameters. This is rather demanding computationally and usually out of interest for lesser streams analyses.

(2) To use the methods of steady flow hydraulics in the determination of longitudinal water surface profiles, corresponding to *N*-year design floods. Indeed, such a method does not provide for solution of an unsteady mode; utilities for a more detailed formulation of flow through control structures are sometimes preferred as an advantage.

In the steady flow option the program can solve both flow regimens: the sub- and the super-critical. The mixed flow option "interfaces" mathematical formulation of both.

– Detailed analysis of the flow through control structures including various hydraulic regimens is also possible, making the model a suitable tool for solving various hydraulic options. It guarantees reliable analyses, mainly in locations where the flow control structures affect hydraulics of the open channel flow. This is also the case of the Jindřichovický creek.

– The steady flow model provides higher values in the solution of the water surface profiles; thus its results are on the side of safety.

Therefore, as already stated, the modelling system HEC-RAS has been used in this study. The detailed program description, the user's manual and hydraulic formulations are available from (HEC-RAS 2005).

Description of the basin

The Jindřichovický creek is a sinistral tributary of the Rotava River at its fluvial kilometer 2.0. Its torrential character is remarkable. The mean slope of the channel is 4%. The catchment area is $F < 35$ km^2, the other factors are: mean altitude $H > 200$ m MSL, the stream slope is rather variable, the flow rates also vary significantly inducing enormous erosion, transport- and bed-load sedimentation, stony up to boulder river bed, fluvial lee wards and hide-outs, trout fish zone (Křovák 2002).

Hydrologic ID number:	1-13-01-114
Total catchment area:	5.964 km^2

Catchment area to US section of river training:
$$1.33 \text{ km}^2$$

Forest coverage:	47%
Length of watershed:	1.62 km
Length of watershed divide:	4.35 km
Watershed shape factor:	$A = 0.653$
Type of the watershed:	Fan-shaped,
hydrographic network not developed	
Torrential coefficient:	$KB = 0.118$

Table 1 lists the *N*-year discharges.

The state of the creek prior to the river training

The upper part of the trained river reaches lines up to the channel lining by a stony pavement at lower parts of the creek. Several steps of almost 2 m height are located in the investigated river reach. The river bed is simply trapezoidal, the width of the bed is 1–1.5 m, and the depth 0.6–1 m. The bed suffers from devastation by extensive stream-side and bottom scouring in several reaches. Several bottom breakdowns and outcrops appear in the channel created by scouring more than 3 m in depth. In these locations, the bed is laid bare down to the bedrock. The river-bed is stony, granularity of carpeting 5–10 cm, and granularity of the boulder strata 20–25 cm. These characteristics were identified by a detailed field investigation of all river reaches. Sizable sandy benches appear in patches of lower flow velocity. The flood plain is overgrown by a mixed forest, the spruce cover dominates. Herbal vegetation with marshland- and ruderal species dominating makes up the sub-canopy of the plain (Křovák 2002).

Hydraulic structures and proposal of water management measures

Being led along the main stream of the basin, most of the proposed thalweg will copy the existing centerline. Due to the considerable lengthwise slope, the channel has only a little tendency to meander. Therefore, only few corrections will be necessary to apply in some river reaches where the irregular trace of the bed will be replaced by free-curvature bedding. The present hollow-ranges of breakdowns will be utilized for the creation of pools. The stabilizing of the pools will be effected using transversal structures of stony riprap and stone packing (rockfill). The modified bed will keep the trapezoidal shape, width in bed 1 m, cross sectional slope 1:1.5 up to the bank lines, stretches with pools excluded. The river bed at

Table 1. Design discharges

N (years)	1	2	5	10	20	50	100
Q (m^3/s)	0.9	1.2	2.2	2.9	3.7	5.4	6.9

the foot and banks up to the height of 0.25–0.30 m will be protected by stone packing (stony rock-fill). Rest of the banks above the stony pavement will be sown.

In effect to considerable gradient in design, the vertical alignment will take into account a number of structures for the slope control of the channel-bed. These will be constructed as wooden sills (shoals), stony weirs (steps) and rock chutes (overruns). Constructional modifications of the structures will provide for fish migration in both directions. This means that the constructional height should not exceed 0.4 m at any cross section and the cross section of the overrun plane should provide for continuous water jet. Natural materials will be exclusively used for all hydraulic measures and their form will cope with the natural formation of the torrent.

Computation scenarios

The evaluation of the channel capacity, the flow velocity, and the proposed stable slope of the river-bed covers 3 choices of the river reaches, all characterized by the effective grain size of d_e = 0.06 m (d_6), 0.10 m (d_{10}), and 0.16 m (d_{16}).

Hydrologic input data

The above-mentioned data set-ups were computed for 2 flow rates of Q_{100} = 6.9 m³/s and Q_5 = 2.2 m³/s.

Geometric input data of the river bed and of the flow control structures

All river reaches including hydraulic structures on the trained stream were calculated as the results of the detailed survey of longitudinal- and transverse profiles. The river profiles are notably impacted by these measures.

Hydraulic input data of the river

The principle hydraulic characteristics of the model are as follows: the roughness coefficient n devised by Manning, and coefficient k according to Strickler. With respect to the original soil of the river bed, to gravel materials, and to building material of the structures, the values of the hydraulic parameters were selected according to HEC-RAS user's manual, determined by the methods of soil mechanics or from the field survey individually for
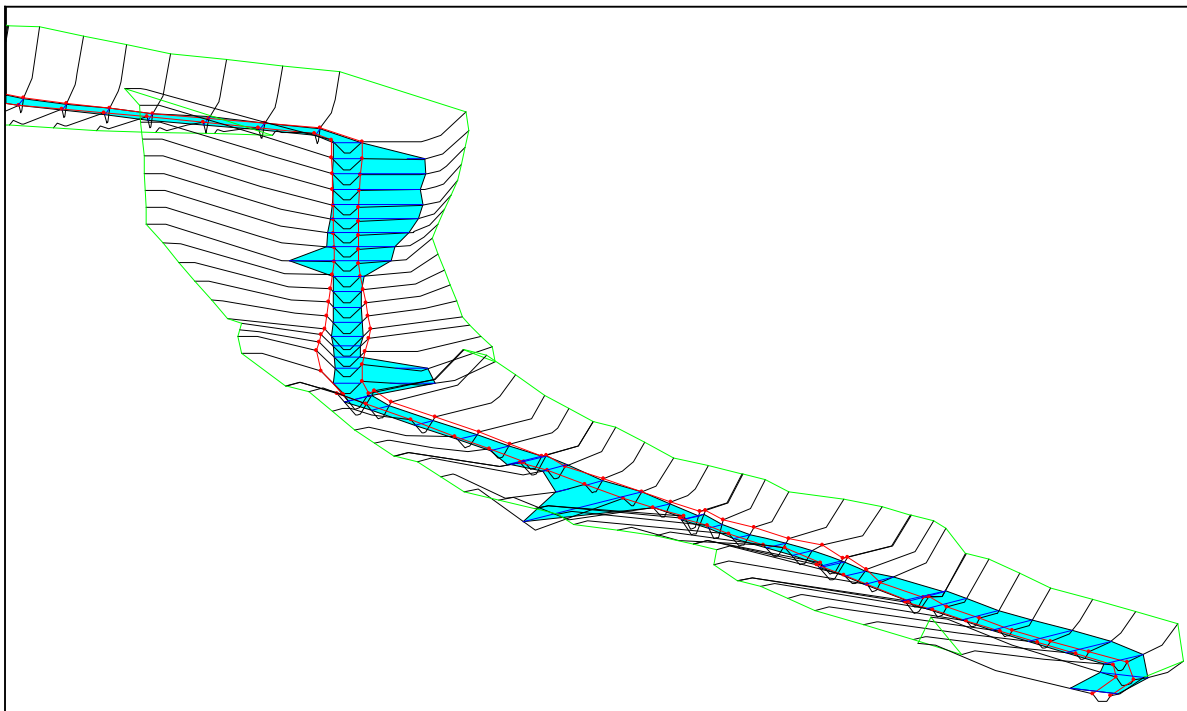

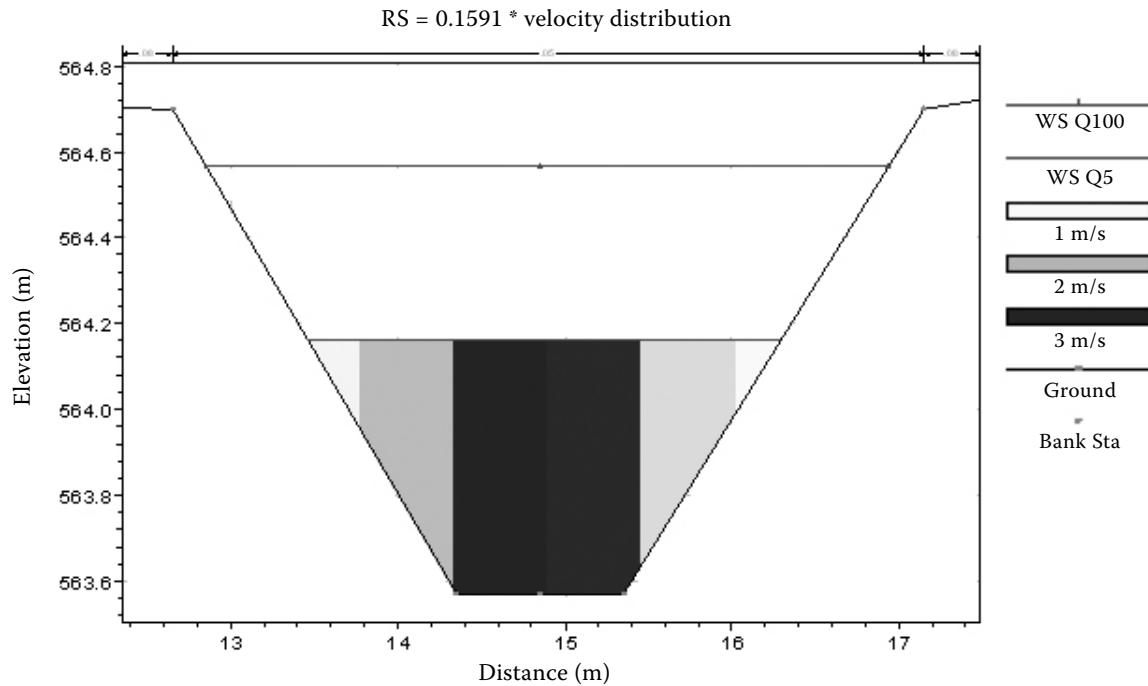Figure 1. Axonometry of the selected reach of the channel, flooded by Q_{100}

Figure 2. Velocity distribution in cross section, flooded by Q_5 and Q_{100}

each cross sectional profile. Granulometric analysis was applied to determine the effective grain size d_e. The *C* coefficient after Novak came in for the torrential rivers Rotava and Rolava (Nováк 1988). The sediment mixture matches the blended torrential bed-load of granite geological formations. Sediment particles are rounded, the shapes of sphere and ellipsoid dominate.

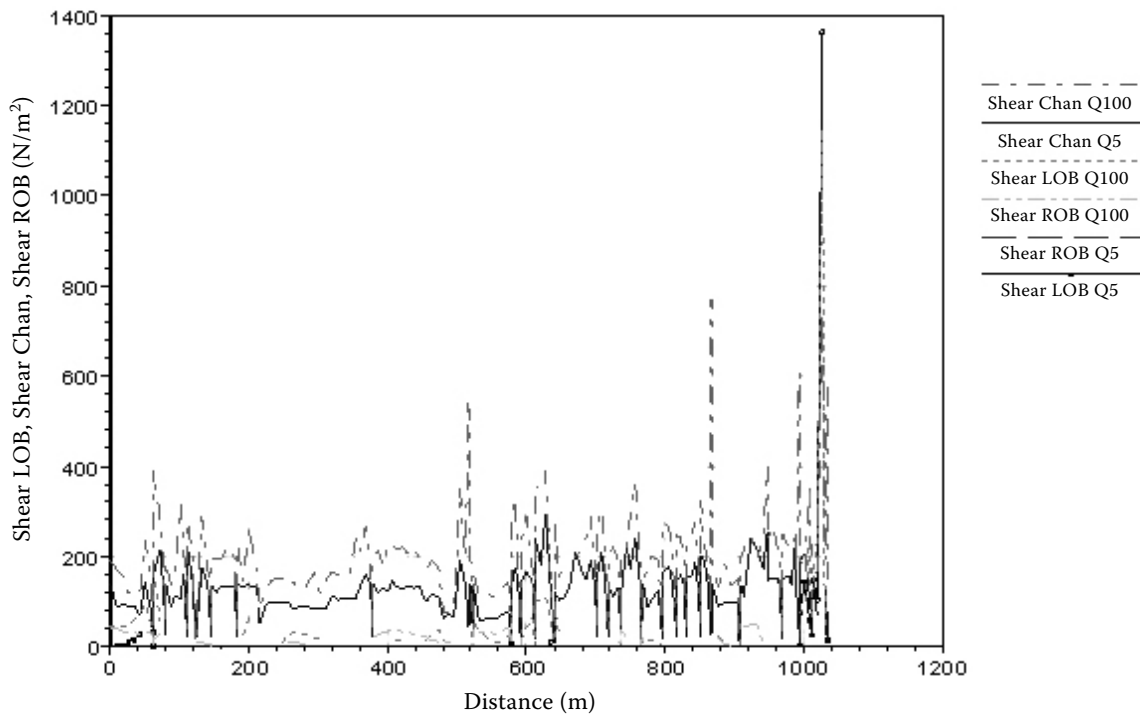Table 2 shows well-arranged hydraulic characteristics of the considered river.

Figure 3. Shear stress distribution in lengthwise profile, flooded by Q_5 and Q_{100}

Table 2. Hydraulic characteristics

Reach No.	d_e	a	n	k	ρ_m	ρ	C	R
1	0.06	18.29	tables	tables	2650	1000	5.58	calculated
2	0.10	16.78	tables	tables	2650	1000	5.58	calculated
3	0.16	15.31	tables	tables	2650	1000	5.58	calculated

RESULTS OF CALCULATIONS

Steady-state calculations under non-uniform flow conditions were performed for three selected river-reaches, all characterized by the effective grain size d_e. The discharge, mean flow velocity, and geometric characteristics of the cross sectional profiles were identified by virtue of the model HEC-RAS. (Figures 1–3) To specify realistic values of the roughness coefficients, the pictorial description from www.hec.usace.army.mil/n-values.index was used.

On the basis of these data the individual equations have been verified and the results summarized into the following pictures and graphs (Figures 4 and 5).

CONCLUSIONS

The basis for the calculation of the stable bottom slope became the classical Shields equation (1) based on the shear stress philosophy. Surprisingly, a good agreement was also shown with the calculation after Manning-Strickler (2). The Eq. (3) of

Novak indicates somewhat higher values of the slope stability. On the other hand, it reflects the features and the origin of the bed-load motion. The difference between the extreme results amounts to 10% (Figures 4 and 5). Such an agreement is influenced both by the selection of the equation and, primarily, by detailed analysis of the sediment load granularity and by a careful evaluation of the roughness coefficients. Such procedure is usually laborious and thus ineffective in engineering practices. Consequently, for the torrent training it is hardly applicable.

Due to the complexity of the processes involved in the water-induced sediment load, the problems of water saturation by the bed-load were totally omitted in this analysis. Until today, the process has not yet been completely examined and thus its use in field is not easily applicable. At a high rate of saturation, part of energy entering the bed-load transport and the scouring may be missing. Unexpectedly, this may become a paradox if applied at higher flow rates. In addition, the design flow rate determination brings further difficulties. The uncertainty in its definition is
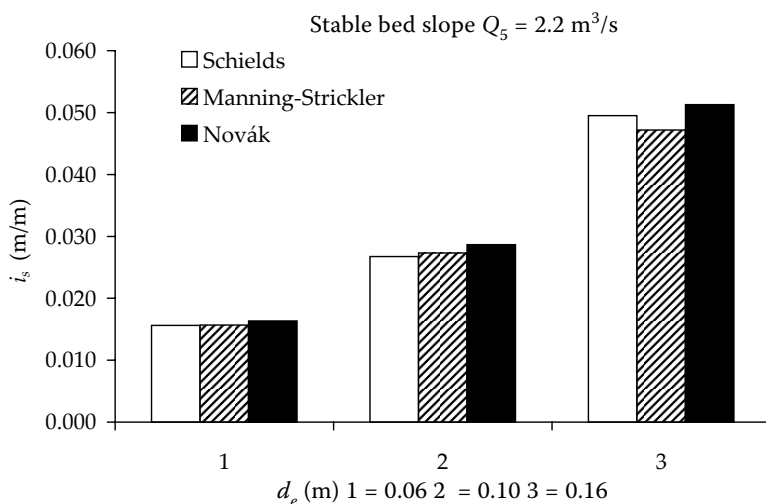

Figure 4. Comparison of stable bed slope; equations of Schields, Manning-Strickler and Novak, discharge Q_5
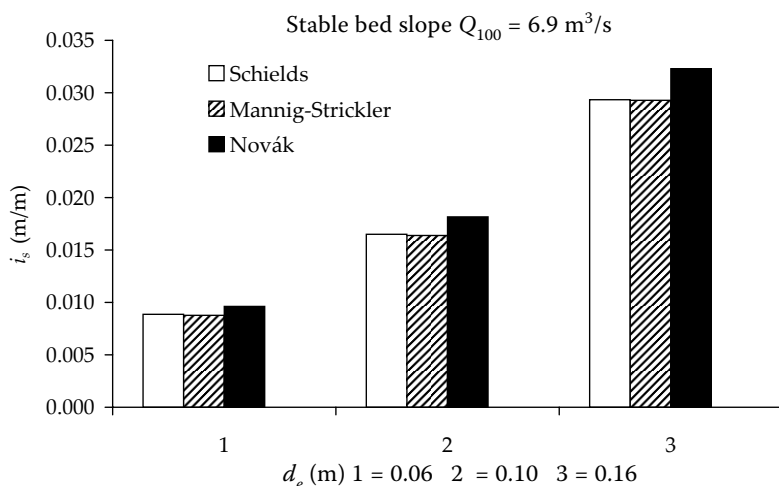
Figure 5. Comparison of stable bed slope; equations of Schields, Manning-Strickler and Novak, discharge Q_{100}

rather complex, particularly in the selection of its "true" value needed for the determination of the optimum stable slope. For the Jindřichovický creek, the centenary flood from 1 km^2 amounts to, e.g., 5.2 m^3/s.

In general, it can be stated that the model accuracy increases with the reliability of granulometric analysis of the bed-load. The more detailed and reliable data exist, the better results of the model can be expected. The optimal determination is questionable from the practical point of view and also from the economic aspects. Therefore, it is with advantage to practice the use of the model categories, where cross section profiles can be easily subdivided in horizontal and in vertical directions. Then spatial distribution of the values of n or of k in any river section can be investigated. Since no comprehensive method still exists for the channel roughness determination, torrent-training designer is forced to use its own experience. The freely distributed documentation on HEC-RAS Web pages has been made available and it becomes a useful starting point for initial estimates of both parameters. In these materials, one can find well verified roughness values for natural rivers, with respect to detailed geometry of the river-bed and to drainage area. The photo-documentation of the representative cases in the United States is also available. It would be therefore highly recommended to work out similar guidelines, at least for the representative Bohemian torrential rivers. Professor Novak has initiated such an effort in the past and it remains just to follow up.

Comment at the end

The last version HEC-RAS 4.0 Beta (2006) includes two new features related to the sediment transport.

Sediment transport/movable bed analyses. This component of the modelling system is intended for the simulation of one-dimensional sediment transport/movable boundary calculations resulting from scour and deposition over moderate time periods (typically years, although applications to single flood events are possible). The sediment transport potential is computed by grain size fraction, thereby allowing the simulation of hydraulic sorting and armouring. The model is designed to simulate long-term trends of scour and deposition in a stream channel that might result from modifying the frequency and duration of the water discharge and stage, or modifying the channel geometry. This system can be used to evaluate the deposition in reservoirs, design channel contractions required to maintain navigation depths, to predict the influence of dredging on the rate of deposition, to estimate maximum possible scour during large flood events, and to evaluate sedimentation in fixed channels.

Sediment Impact Analysis Methods (SIAM). SIAM is a sediment budget tool that compares annualized sediment reach transport capacities to supplies and indicates the reaches of overall sediment surplus or deficit. SIAM is a screening level tool to compute rough, relative responses to a range of alternatives, in order to identify the most promising alternatives (which should then

be modelled in more detail). The algorithms in SIAM evaluate the sediment impact caused by local changes in the system from the sediment continuity perspective. The results map potential imbalances and instabilities in a channel network and provide the first step in designing or refining remediation. Users can begin with the existing geometry and flow data and develop a set of sediment reaches with unique sediment and hydraulic characteristics. The SIAM program will then perform sediment transport capacity computations to determine potential imbalances and instabilities in a channel network. SIAM does not predict intermediate or final morphological patterns and does not update cross sections, but rather indicates trends of locations in the system for potential sediment surpluses or deficits. The results can be used to design or refine remediation efforts in the system.

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Corresponding author:

Ing. František Křovák, CSc., Česká zemědělská univerzita v Praze, Fakulta lesnická a environmentální, katedra biotechnických úprav krajiny, Kamýcká 129, 165 21 Praha 6-Suchdol, Česká republika
tel.: + 420 224 382 138, + 420 224 382 148, e-mail: krovak@fle.czu.cz