Approximation of Subsurface Drainage Discharge by De Zeeuw-Hellinga Theory and its Verification in Heavy Soils of Fluvial Landscape of the Cerhovice Brook

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Abstract: The subsurface drainage discharge is one of the most important indicators of the impact of the drainage systems on the water management. The procedure adopted in this study is based on the application of the De Zeeuw-Hellinga theory to derive the final expression for the estimation of the value of the subsurface drainage discharge. A simple analytical approximation of the Bussinesq's Equation was used to verify theoretically the validity of the De Zeeuw-Hellinga assumptions and to confirm the correctness of other corresponding processes. The formulas describing the subsurface drainage discharge were derived in the conditions of the unsteady state subsurface flow to drains. These conditions included the approximately horizontal impervious layer and the Dupuit's assumptions and Darcy's law. No recharge to the groundwater table was realised during the drainage testing. The applicability of the De Zeeuw-Hellinga formula and the accuracy of the analytical approximation of the subsurface drainage discharge by the Bussinesq's Equation were verified by the real field measurements on the heavy soils of the experimental watershed area of the Research Institute for Soil and Water Conservation (RISWC) Prague-Zbraslav, Czech Republic. The same data were successfully used also for the confirmation of the accuracy of the method for the derivation of a simple analytical approximation of the subsurface total drainage quantity. It was demonstrated that this approximation of the subsurface drainage discharge by De Zeeuw-Hellinga theory could satisfactorily serve in the area of water engineering practice as an elementary tool for the immediate estimation of the values of the subsurface drainage discharges from the pipe drainage systems in the saturated porous environment. The advantage of this approximation is particularly the minimum amount of the input data, e.g. the basic soil hydrology data and drainage system basic design parameters. The sphere of the use of the De Zeeuw-Hellinga equations is certainly very wide. The verifications of the field test results and measurements demonstrated that the possibilities of applications and their perceived benefits to the user can be fulfilled.

Keywords: subsurface pipe drainage system; subsurface drainage discharge; De Zeeuw-Hellinga theory; Bussinesq's Equation; unsteady drainage flow conditions

The importance of the drainage system in the water management, particularly the subsurface pipe drainage system, is indisputable. For large-scale territorial units in the region of South-East Asia, Africa, India, or China, the existence of the subsurface pipe drainage systems is a neces-

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sity. This applies particularly when their specific natural conditions bring about permanent waterlogging (ILRI/ALTERRA/UR 2000; Ritzema 1994).

A typical example of the importance of the drainage policy in Europe are such countries as the Netherlands, Lithuania or Denmark, where the ratio of the drained area to the total area of the country is 0.72, 0.4, and 0.37 respectively (RISWC 2006).

Another example of the land drainage is offered by the UK. A large-scale land drainage scheme can be found in the Crossens catchment at Southport, situated in Lancashire, to the north of Liverpool. The Crossens catchment contains about 145 km^2 of low-lying land covered mostly with wet lowland peat. The necessary drainage policy is based on the upper and lower drainage systems, both systems joining at the Crossens Pumping Station at Southport, which pumps water out of the catchment to the sea (Harris *et al.* 2004; Rosolova 2006)

Regarding the drainage systems, one of the many important contemporary problems of the water management in the Czech Republic is the long-lasting lack of proper maintenance of the older subsurface drainage pipe systems in the agricultural and riverine (fluvial) landscape and the estimation of its hydraulic efficiency and cost-effectiveness.

To solve these problems, the Research Institute for Soil and Water Conservation (RISWC) Prague-Zbraslav, Czech Republic, the Czech Academy of Agricultural Sciences – Water Management Section, and the Czech Committee of ICID (International Commission on Irrigation and Drainage) arranged a workshop on Drainage of Agricultural Lands in the Context of Cultural Landscape on November 3, 2005 in Prague (RISWC 2005).

Shortly thereafter, the riverine landscape was flooded (Kovář & Štibinger 2008), and while the excess runoff left the catchment, the soil profile remained fully saturated. The high soil water content is caused by the raised water table levels. Under such conditions, the subsurface pipe drainage system as a part of the local water management structures can optimise the groundwater regime by its hydraulic function. This ensures an effective reduction of the negative impacts of floods.

From the hydrologic point of view, it seems that the current dynamics of the climate is characterised by a great number of extremes, e.g. typhoons, tropic storms, massive short-term rainstorms, and other similar natural phenomena. The negative impacts of these hydrological events very often result in severe flooding.

Some of the territorial units not only in Europe (e.g. the Netherlands), but, for example, also in the Pacific Ocean, are situated under the sea level. Therefore they have to face the permanent impact of such conditions on the water pressure and seepage.

The floods, which struck the Czech Republic in 1997, 2002 and 2006, positively demonstrated the importance of and the reason for the drainage systems and infiltration ability of the surface layers in the landscape.

The subsurface drainage discharge, generated by the presence of the subsurface pipe drainage systems in saturated soils under the unsteady state drainage flow, comes as one of the cardinal indicators of the drainage hydrology and it has to be taken into consideration within the possible solutions of the water management problems.

Should the subsurface pipe drainage system have sufficient impact on the water table and hence improve the land properties for the environmental protection, the design of the parameters of such a system would have to be based on the analysis of the subsurface drainage discharge. This discharge is the essential indicator of the impact of the field drainage on the water regime.

This paper reviews the procedure of the application of the De Zeeuw-Hellinga theory for obtaining the final expression for the estimation of the value of the subsurface drainage discharge.

The methodology of a simple analytical approximation of the subsurface drainage discharge by Bussinesq's Equation to verify theoretically the validity of the De Zeeuw-Hellinga assumptions and to confirm the accuracy of the other corresponding processes is also explained in this report.

The formulas of the subsurface drainage discharge were derived with the following assumptions: the subsurface flow to drains occurred only when an approximately horizontal impervious layer was present and the Dupuit's assumptions and Darcy's law were valid.

The correctness and applicability of the analytical approximations of the subsurface drainage discharge, which were shaped into a single equation, were verified by the real field measurements on heavy soils of the experimental watershed area of the Research Institute for Soil and Water Conservation (RISWC) Prague-Zbraslav, Czech Republic (Soukup *et al.* 2000).

The data from the experimental field of the RISWC Prague-Zbraslav, which were used for the verification of the results of the research mentioned above, successfully served also for the confirmation of the accuracy of the derivation of a simple analytical approximation of the subsurface total drainage quantity in non-steady drainage flow (Štibinger 2003).

The use of the field data collected for this study was twofold. They were used to test and verify the principles of the derivation of various characteristics of the drainage hydrology of subsurface drainage discharges obtained by De Zeeuw-Hellinga theory and by analytical solution of the Boussinesq equation on one hand, and the subsurface total drainage quantity derived by Štibinger (2003) on the other.

The derivative equations of the subsurface drainage discharge can serve as a good elementary tool of water engineering practice for an immediate estimation of the values of the subsurface drainage discharge from the subsurface pipe drainage systems in saturated soils and under the non-steady state drainage flow. Also, it can serve as a tool for the analysis of the subsurface drainage discharge hydrology.

The principal part of the procedures presented in this paper is the inseparable strand of the research of the Ministry of Agriculture of the Czech Republic no. NPV-MZe 2005. VRK1/TP3-DP6 (1G57040), Methodology of Option of Optimal Variant of Flood Protection and Erosion Control in Landscape (Kovář & Štibinger 2008), which is solved by Czech University of Life Sciences Prague, Faculty of Environmental Sciences, Department of Land Use and Improvement.

MATERIALS AND METHOD

Study area

To verify this numerical experiment, the measured values of the subsurface drainage discharges were used. These data were obtained from the field experimental area of RISWC, Prague-Zbraslav. An area of approximately 40 ha of drained heavy clay soils located in the experimental watershed was selected for the research purpose of the evaluation of the subsurface drainage policy impact.

The RISWC, Prague-Zbraslav experimental field forms a part of the watershed area of the riverine landscape of the Cerhovice Brook. The entire watershed area is situated in the Central Czech Upland with the altitude of 350–500 m a.s.l. and with the long-term annual average precipitation of 560 mm. All soil layers at this location have a low permeability and the depth to the impervious barrier is approximately 0.9 m bellow the soil surface. The parent rock is formed from shale.

The part of the experimental field area used for the verification is drained by the subsurface horizontal parallel systematic drain system with the drain spacing L = 11 (m), average of drain depth h_d = 0.75 (m), and diameter of the lateral drain r_0 = 0.06 (m). This drainage system represents a typical example of the shallow subsurface drainage system of heavy soils with its low hydraulic saturated conductivity. During the process of drainage, no recharge to the water table was monitored.

The drainage discharge data, which were used for the verification, were chosen from June 2000 through to July 2001. The start of the time series of the daily measured values of the subsurface drainage discharges was May 7, 2001, and the end of the non steady-state drainage processing was around May 29, 2001, when the subsurface drainage rate dropped under the measurable value.

De Zeeuw-Hellinga theory

The derivation of the subsurface drainage discharge q (M/T) in non-steady state drainage flow was performed by means of the De Zeeuw-Hellinga theory (De Zeeuw & Hellinga 1958; Štibinger 2006). The period with a non-uniform distribution of the drainage recharge R (M/T) was divided into time intervals of equal lengths (minute, day, week, month). Then the subsurface drainage discharge q (M/T) could be calculated gradually, step by step, or interval by interval, for every time interval. Symbol M (respective T) represents the unit of length (respective time unit).

De Zeeuw and Hellinga found out (Ritzema 1994) that, if the drainage recharge R (M) in each time interval (T) is assumed to be constant, the change in the drainage discharge is directly proportional to the excess drainage recharge ($R - q$) (M) in the respective time interval.

The constant of proportionality is De Zeeuw-Hellinga drainage intensity factor $a(T^{-1}) = \pi^2 KH/(P_d L^2)$, which depends on the parameters of the pipe drainage system, on the hydraulic properties of the soil environment, the position of the water table level, and the position of the impervious layer (Dieleman & Trafford 1976).

where:
a – De Zeeuw-Hellinga drainage intensity factor (T^{-1})
K – hydraulic saturated conductivity of the drained soil environment (M/T)
H – average depth of aquifer (M)
P_d – drainable pore space of the drained soil (–)
L – drain spacing (M)

In this case, the drainage intensity factor a (T^{-1}) also expresses the hydraulic efficiency of the subsurface pipe drainage system.

The initial equation describing the De Zeeuw-Hellinga theory explained above can be expressed as:

$$\frac{\partial q}{\partial t} = a(R - q) \qquad (1)$$

The De Zeeuw-Hellinga's expression in the form of equation (1) actually constitutes an ordinary differential equation where the drainage factors a (T^{-1}) and R (M) are the constants and q (M) is the unknown function of variable t. The equations of this type can be solved by the separation of variables. The final form of the De Zeeuw-Hellinga drainage theory is achieved:

$$q(t) = q(t - dt)e^{-adt} + R(1 - e^{-adt}) \qquad (2)$$

or

$$q_t = q_{t-1}e^{-adt} + R(1 - e^{-adt}) \qquad (3)$$

The detailed description of the analysis and solution of the De Zeeuw-Hellinga drainage theory was presented by Štibinger (2006). If no recharge of the water table is recorded during the water table recession via the subsurface pipe drainage system (e.g. following the rainfalls, irrigations, heavy rains or floods), then $R = 0$. Eq. (11) can be written as a formula to approximate the subsurface drainage discharge:

$$q_t = q_{t-1}e^{-adt} \qquad (4)$$

The first term q_{t-1} of Eq. (4) has the dimension of the drainage rate, drainage intensity (M/T) and the second term e^{-at} are dimensionless.

With respect to the constant time interval dt, the initial value of the subsurface drainage discharge at the beginning of the drainage process will be approximated from the steady state conditions. For example Ritzema (1994) used Hooghoudt's Equation in this sense.

Using the calculations given in Eq. (4) and with the knowledge of the basic subsurface drainage system parameters and soil hydrology characteristics (K, P, H), it is possible to evaluate on step-by-step basis the subsurface drainage discharge q_t (M/T) in any time interval dt (practically in certain time $t > 0$).

The De Zeeuw-Hellinga's method for the calculation of the subsurface drainage discharge q_t (M/T) is very useful and has a wide range of use. For example, if any recharge R (M/T) in any time interval dt is present, the subsurface drainage discharge $q(t)$ (M/T) will be estimated by Eq. (3). Similar procedures, i.e. using the method of superposition, were presented by Kraijenhoff (1958) and Maasland (1959). Those methods were simplified by Dieleman and Trafford (1976) for engineering drainage practice.

A practical example of the application of the De Zeeuw-Hellinga's theory to approximate landfill drainage discharge was shown by Štibinger (2006).

Analytical approximation

For the saturated non-steady state one-dimensional horizontal flow $q(x)$ according to the Dupuit's assumptions, Darcy's equation can be written as:

$$q(x) = -h(x,t) \times K \frac{\partial h(x,\, t)}{\partial x} \qquad (5)$$

where:
$q(x)$ – intensity of the saturated non-steady state one-dimensional horizontal flow (M^2/T)
$h(x,t)$ – head of the free water table level in the soil porous environment in an arbitrary horizontal distance x (M) from the origin at an arbitrary time $t > 0$; (M)

The change in water storage per unit surface area in an infinitely small period of time is described by the equation of continuity:

$$-\frac{\partial q(x)}{\partial x} = P_d \frac{\partial h(x,\, t)}{\partial t} \qquad (6)$$

By substitution Eq. (5) with Eq. (6), a non-linear, partial differential equation of the second-order will be obtained:

$$\frac{\partial(h(x,t) \times K[\,\partial h(x,\, t)/\partial x])}{\partial x} = P_d \frac{\partial h(x,\, t)}{\partial t} \qquad (7)$$

The non-linearity demonstrates the first part of Eq. (7): $\frac{\partial(h(x,t) \times K\,[\partial h(x,\,t)/\partial x])}{\partial x}$. By the approximation $h(x,t)$ (M) = H (M) constant in the first part of this equation, where H (M) = constant and represents the average depth of the aquifer, Eq. (7) can be formulated as:

$$HK\frac{\partial^2 h(x,t)}{\partial x^2} = P_d\frac{\partial h(x,\,t)}{\partial t} \tag{8}$$

Eq. (7) and (8) are also known as the Boussinesq's Equation (Boussinesq 1904) and serve as very good tools not only for the description of the non steady-state groundwater flow in a general form, but also for the analysis of subsurface drainage processes (Dumm 1954; Glover 1964; Dieleman & Trafford 1976; Sagar & Preller 1980; Ritzema 1994; Štibinger 2003).

In this case, it is assumed that no recharge to the groundwater table occurs. The unsteady-state saturated groundwater flow to the subsurface pipe drainage system, without any recharge to the water table, is accurately described by these equations.

The linearisation of Eq. (7) expressed by Eq. (8) applies mainly in the case of deep impervious barriers. It assumes that where H (M) converges to Hooghoudt's equivalent depth l' (M), the height of the water table above the level of drain can be neglected (Ritzema 1994).

In the following case, the average depth of the aquifer H (M) was approached as

$$H\,(\mathrm{M}) = l' + \frac{h_0}{4}$$

where:

h_0 – initial water table level (M) at time $t = 0$

The simplified analytical solution of the linearised Boussinesq's Eq. (8) can be presented in the shape of $h(x,t) = \frac{4h_0}{\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}e^{-n^2at}\sin(\frac{n\pi x}{L})$. If $a.t > 0.2$, the second and following terms of this expression can be neglected, the formula is reduced to:

$$h(x,t) = \frac{4h_0}{\pi}\,e^{-at}\sin\left(\frac{\pi x}{L}\right) \tag{9}$$

which is the starting formula for the analytical approximation of the subsurface total drainage quantity $Q(t)$ (M). Štibinger (2003) derived this parameter into the equation:

$$Q(t) = h_0P_D\left(1 - \frac{8}{\pi^2}e^{-at}\right) \tag{10}$$

Finally, by the differentiation of the right part of Eq. (10) in time t the final expression of the subsurface drainage discharge $q(t)$ (M/T) at the any time $t > 0$ is obtained in the form of:

$$q(t) = (8h_0P_D.a/\pi^2).e^{-at} \tag{11}$$

Eq. (11) expresses the value of the subsurface drainage discharge $q(t)$ (M/T).

Similarly as in the case of Eq. (4), the first term $(8h_0P.a/\pi^2)$ of Eq. (11) has the dimension of the drainage rate, drainage intensity (M/T) and the second term e^{-at} are dimensionless. Alongside, the equation is, of course, identical with the second term of Eq. (4).

In this way the validity of the De Zeeuw-Hellinga theory and the accuracy of the method of the derivation were theoretically verified. This leads to the derivation of the final form of Eq. (4).

The way of the analytical solution of the subsurface drainage discharge $q(t)$ (M/T), which is described in the final form by Eq. (11), is exploitable enough. This process permits to approximate the subsurface total drainage quantity $Q(t)$ (M) at any time $t > 0$, in this case by Eq. (10).

From the Darcy's Law, Ritzema (1994) derived the subsurface drainage discharge as:

$$q(t) = h_0(8KH/L^2).e^{-at} \tag{12}$$

By substituting $a(T^{-1}) = \pi^2KH/(P_dL^2)$ it becomes obvious, that the first term $h_0(8KH/L^2)$ of Eq. (19) is identical with the first term $(8h_0P.a/\pi^2)$ of Eq. (11). The procedure explained above affirms and supports the correctness of the method of the analytical solution of the Boussinesq Equation, leading to the final form of Eq. (11).

The linearisation of the Boussinesq Equation is valid for homogenous isotropic porous soils with a continuous groundwater table and for small changes of H value in the fully saturated soil environment. Those conditions, in the case of the drained heavy clay soils located in the experimental watershed of the Cerhovice Brook (RISWC, Prague-Zbraslav), were not fulfilled.

That is why the use and application of the linearisation of the Boussinesq Equation is speculated on in this case just only as a hypothesis. The analysis and results of this use do not fall into the main goals of this research.

RESULTS AND DISCUSSION

Measured and calculated values

The accuracy of the De Zeeuw-Hellinga's model application represented by Eq. (4) and the validity of the use of the analytical solution represented by Eq. (11) were verified by measured real values of the subsurface drainage discharges. These were obtained from the field experimental area of the Research Institute for Soil and Water Conservation (RISWC) Prague-Zbraslav, Czech Republic (Soukup *et al.* 2000).

The calculated values of the subsurface drainage discharges obtained by the application of Eq. (4) and Eq. (11) were compared with the daily measured data from the RISWC Prague-Zbraslav experimental watershed mentioned above.

The area of 40 hectares of heavy soils with a low hydraulic conductivity drained by a systematic subsurface pipe drainage system was selected and established during years 1976/77 by the RISWC Prague-Zbraslav. The main purpose was to estimate the impact of the drainage policy and particularly the force of the subsurface pipe drainage system on the water regime in the agriculture landscape.

This part of drainage represents a typical example of the shallow subsurface drainage system (see Figure 1), which comprises the drain spacing L = 11 (m), average of the drain depth h_d = 0.75 (m), and diameter of the lateral drain r_0 = 0.06 (m) (Štibinger 2003).

The soil hydrology characteristics of the drained soil layers were measured in the terrain and verified in the laboratory using the "undisturbed" core samples. The approximation of the hydraulic saturated conductivity values was executed by the principles and applications of the single auger-hole method with Ernst's evaluation (Van Beers 1970), partially by the inversed single auger-hole method (Ritzema 1994) and double-ring infiltration method (Kutílek & Nielsen 1994). The drainable pore space (effective porosity) was approximated from the soil water retention curves (SWRC) using van Genuchten's theory (van Genuchten & Nielsen 1985) with equivalent pore radius distribution.

The non-homogenous environment of heavy clay soils with a low permeability was represented by one value of hydraulic saturated conductivity K = 0.075 (m/day), by the drainable pore space (effective porosity) P_d = 0.015 (–), by the thickness of the impermeable soil profile 0.90 (m). The groundwater table was not continuous, the hypothetic representative initial water table level was h_0 = 0.50 (m).

The initial drainage and hydraulic calculations show the value of l' = 0.15 (m) (lateral drains are situated very close to the impervious layer), the value of $H = l' + (h_0/4)$ = 0.275 (m), and indicate the value of the drainage intensity factor $a = (\pi^2.K.H)/(L^2.P_d)$ = 0.112 (day^{-1}).

The groundwater table levels above the drain pipes are periodically measured by the piezometers. During the water table recession (subsurface drainage processing) no precipitation, heavy rains, irrigation, or floods were monitored to the groundwater table level.

That means that the process of the subsurface saturated unsteady-state flow to the drains was not shaped by any recharge to the groundwater table.

The period from which the time series of the subsurface drainage discharge for the verification were selected, started at the beginning of May 2001 and proceeded to the end of the month, when the unsteady state drainage process terminated.

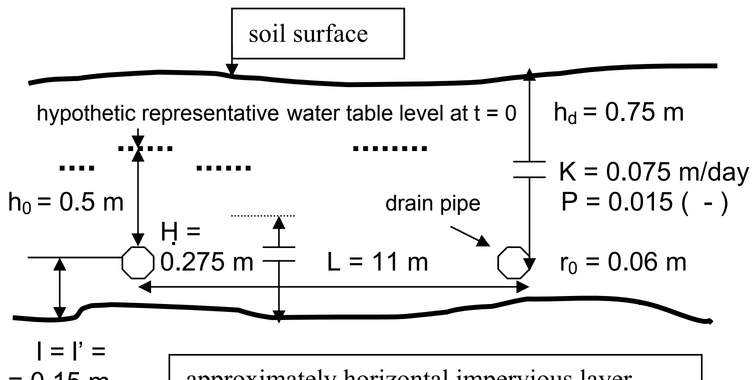

Figure 1. Shallow subsurface drainage system parameters in non-steady state conditions of the RISWC Prague-Zbraslav experimental field

Table 1. Measured and calculated daily values of the subsurface drainage discharge from the RISWC Prague-Zbraslav experimental field

Days	Subsurface drainage discharge measured values (mm/day)	Subsurface drainage discharge De Zeeuw-Hellinga values (mm/day)	Subsurface drainage discharge calculated values by Eq. (11) (mm/day)
1	0.95	0.89	0.61
2	0.78	0.80	0.54
3	0.63	0.71	0.49
4	0.53	0.64	0.43
5	0.49	0.57	0.39
6	0.42	0.51	0.35
7	0.38	0.45	0.31
8	0.35	0.41	0.28
9	0.29	0.36	0.25
10	0.26	0.32	0.22
11	0.23	0.29	0.20

The real measured daily values of the subsurface drainage discharge (mm/day), from the selected period specified above, are shown in the second column of Table 1.

The results of the calculations of the daily values of the subsurface drainage discharge (mm/day) obtained by Eq. (4), based on the De Zeeuw-Hellinga drainage theory, are shown in the third column of Table 1.

Daily values of the subsurface drainage discharge (mm/day) calculated according to Eq. (11) are shown in the last column in Table 1. These come from the simplified analytical solution of the linearised Boussinesq's equation.

The time series of all the measured and calculated data are presented graphically in Figure 2.

DISCUSSION

The daily values of the subsurface drainage discharge measured at the RISWC Prague experimental field, the values of the subsurface drainage discharge calculated by De Zeeuw-Hellinga model (4) and the values of the subsurface drainage discharge calculated by Eq. (11) were compared (Table 1 and Figure 2).

Graph in Figure 1 shows that the trends of the curves of the measured and calculated values are identical, although certain differences are apparent.

The course of the time series of the tested values is monotone, exponential, evidently decreasing and corresponds with the real drainage processes very well.

As obvious in Figure 2, it appears that especially at the beginning of the tested period the values of the subsurface drainage discharge obtained by the De Zeeuw-Hellinga model approximate the process far better then the values of the subsurface drainage discharge (calculated according to Eq. (11)).

Approximately after three and a half days of the process, the values obtained by the De Zeeuw-Hellinga theory are evidently higher then the real values, while the values estimated by Eq. (11) are clearly smaller (see Figure 2).

Particularly during the first days of the drainage process, the deviations (errors) of the values of the subsurface drainage discharge calculated by Eq. (11) from the measured real values were rather high: On the first day it was 35.2%, on the second day 25.2%, and on the third day 14.7% (see Table 2).

Dieleman and Trafford (1976) offer an explanation for this discrepancy. They discovered that the validity of Eq. (11) is defined from a certain point of time T_p, where the relation between the subsurface drainage discharge and the decrease of the water table level is constant. The value of T_p can be estimated from the expression $T_p = 0.4/a$ (days).

In this case $T_p = 0.4/0.112 = 3.57$ days, which means that Eq. (18) should be used just from the time of 3.6 days. In this way are explained the

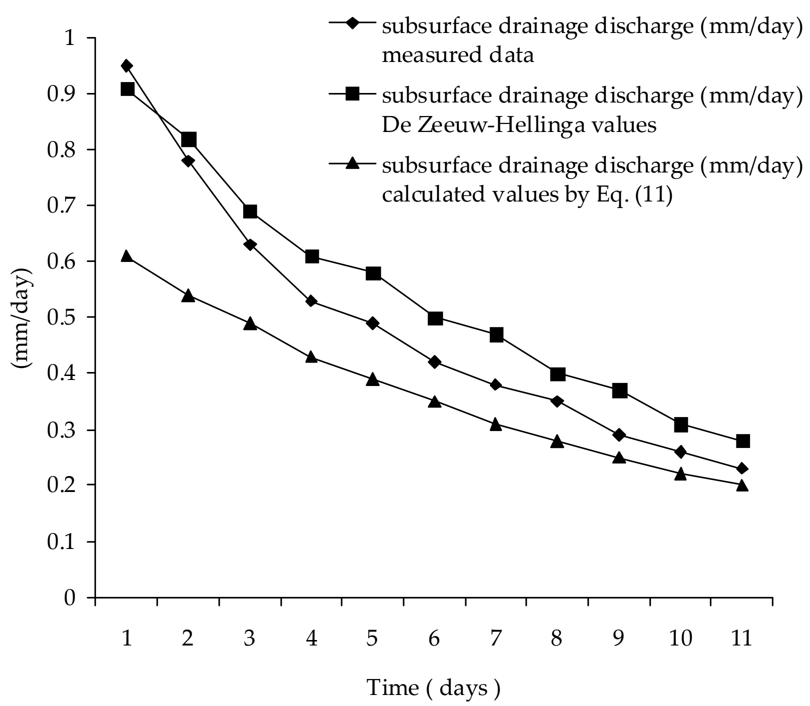

Figure 2. Measured and calculated daily values of the subsurface drainage discharge from the RISWC Prague-Zbraslav experimental field

more significant errors of the subsurface drainage discharge values, calculated by Eq. (11) in the first three days of this drainage process.

According to Mls (1988), for the saturated non-steady state groundwater flow with the use of the Dupuit's assumptions and Darcy's Law, the following theory applies: At the border (for example a bank or a drain pipe) and for the time on-coming (converging) to zero, the relatively extremely high values of the velocities, drainage discharges, and outflows are generated.

The graph in Figure 3 clearly shows that the course of the absolute magnitude of the differences generated by the use of Eq. (11) (absolute magnitude from the daily values of the measured drainage discharge minus $q(t)$ calculated by Eq. (18)) is monotone, slightly decreasing. The differences are directly proportional to the values of the subsurface drainage discharge. The smaller the value of the subsurface drainage discharge, the smaller the difference (error). From the time $t = 3.57 = T_p$ (day) the absolute magnitude of the differences varies between 0.1 (mm/day) and 0.03 (mm/day), which means between 10.5% and 3.1%.

To generalise the results of the analysis of the terrain and numerical experiments by the use of linearisation of the Boussinesq Equation, the research projects of RISWC, Prague-Zbraslav from the experimental area Zaluzi from 1972–1992 will be analysed in the next stage of this research.

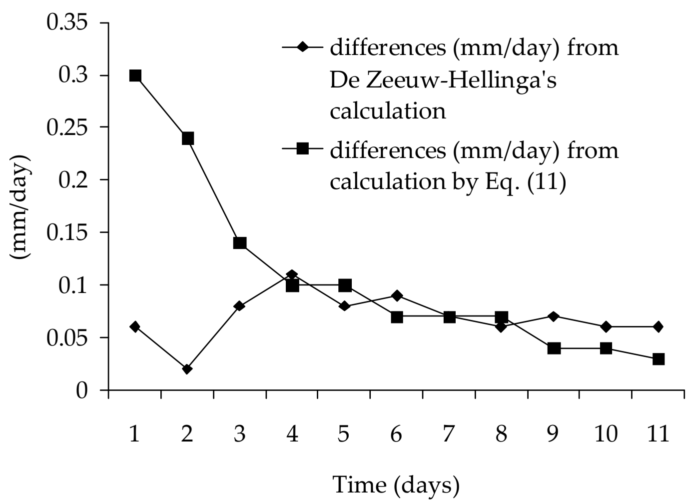

Figure 3. Course of absolute magnitude of differences from De Zeeuw-Hellinga's applications and from calculations by Eq. (11), RISWC Prague-Zbraslav experimental field

Table 2. Differences between measured and calculated (by De Zeeuw-Hellinga model and by Eq. (11)) daily values of the subsurface drainage discharge from the RISWC Prague-Zbraslav experimental field

Days	Differences of De Zeeuw-Hellinga absolute magnitude (mm/day)	Differences (%)	Differences of Eq. (11) absolute magnitude (mm/day)	Differences (%)
1	0.06	6.3	0.34	35.8
2	0.02	2.1	0.24	25.2
3	0.08	8.4	0.14	14.7
4	0.11	11.5	0.10	10.5
5	0.08	8.4	0.10	10.5
6	0.09	9.5	0.07	7.3
7	0.07	7.3	0.07	7.3
8	0.06	6.3	0.07	7.3
9	0.07	7.3	0.04	4.2
10	0.06	6.3	0.04	4.2
11	0.06	6.3	0.03	3.1

The time series of the absolute magnitude of the differences between the daily values of the measured subsurface drainage discharge and the values of q_t calculated by De Zeeuw-Hellinga theory is also shown in the graph in Figure 3. Over the entire observed period, the course of the differences fluctuates, from the fourth day it slightly decreases, and the absolute magnitude of the differences varies between 0.11 (mm/day) and 0.02 (mm/day), i.e. between 11.5% and 2.1% (see Table 2 and Figure 2).

From the fourth day of the drainage process, the course of the differences from De Zeeuw-Hellinga applications is almost identical with the course of the differences from the calculation by Eq. (11).

It is perceived that the final Eq. (4), derived on the basis of the De Zeeuw-Hellinga theory, can serve as a good tool for the approximation of the subsurface drainage discharge over the whole course of the drainage process, while Eq. (11), which is more known and used, is suitable from a certain time of the drainage process. In this case it is from the time t = 3.6 day.

CONCLUSIONS

It is indisputable that the subsurface drainage discharge falls within the most important indicators of the subsurface pipe drainage system and all drainage hydrology. The correct estimation of this characteristic plays the key role in the drainage policy and can serve not only for the evaluation of the impact of the existing subsurface drainage system, but also for the design of the parameters of a new one.

The accuracy of the De Zeeuw-Hellinga's method in the general form was theoretically confirmed by the results of the analytical solution of the Boussinesq's Equation.

By the daily measured real values of the subsurface drainage discharges from the experimental field of the RISWC Prague, it was demonstrated that Eq. (4), as a result of the De Zeeuw-Hellinga theory, is an acceptable tool to approximate the subsurface drainage discharge under the unsteady state drainage flow in heavy soils.

The applicability of the De Zeeuw-Hellinga's assumptions in the highly permeable porous environment was controlled in the field of the landfill hydrology by the analysis of the measured data of the landfill leachate rate from the internal landfill drainage system at the Osecna Landfill in the Czech Republic (Štibinger 2006).

A great privilege of the De Zeeuw-Hellinga Eq. (4), as compared to Eq. (11), consists in the possibility of its use also and especially at the very beginning of the drainage process.

Eq. (11) or other formulas derived on the basis of the analytical solution of the Boussinesq Equation do not offer this possibility. Such equations can be used only from a certain time t (Dieleman & Trafford 1976).

This procedure (policy), of course, is not usable in the case of the application of Eq. (11) or in the case of the use of other formulas derived on the basis of the analytical solution of the Boussinesq Equation, which can be used just from a certain time, t only (Dieleman & Trafford 1976).

Under the umbrella of the Ministry of Agriculture Czech Republic, the Department of Land Use and Improvement, Faculty of Environmental Sciences, Czech University of Life Sciences Prague was commissioned to start a research no. NPV-MZe 2005. VRK1/TP3-DP6 (1G57040), named Methodology of Option of Optimal Variant of Flood Protection and Erosion Control in Landscape (Kovář & Štibinger 2008).

The practical engineering applications of all forms of the De Zeeuw-Hellinga assumptions with other procedures are used just in this project. The aim is to estimate the values of the subsurface drainage discharge in a concrete area of interest in the landscape.

Despite the necessary use of models such as DRAINMOD (Skaggs 1999), SWAP (Dam 2000), MODFLOW in complex cases of the drainage policy (unsaturated zone, cracked soils, transient drainage processes), the application of the algorithms and the calculators in the style of Eq. (3) and (4) has its own not negligible advantages.

De Zeeuw-Hellinga drainage model approximation yields slightly higher values of the subsurface drainage discharges then are the actual data. This makes the application of its results useful in the design of effective subsurface pipe drainage systems. It can also serve as a simple and suitable engineering tool for the immediate estimation of the subsurface drainage discharge value, which requires only minimum input information (e.g. the basic soil hydrology data and drainage system basic design parameters).

Last but not least, the results of the De Zeeuw-Hellinga theory in the single final form, as represented by Eqs (3) and (4), allow their use (can be used) in an inverse situation.

This means that the equations can be used not only for the estimation of the impact of the subsurface field drainage systems on the water regime in the landscape, but also for the verification of the design parameters of the existing subsurface drainage systems, or for determining a new one.

The sphere of the use of the De Zeeuw-Hellinga equations is very wide, all the verifications of the field test results and measurements prove that the possibilities of applications and their benefits to the user, as mentioned above, can be fulfilled.

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