

Multi-objective optimisation and synergistic mechanisms of expansive soil improvement using organic fertiliser, slow-release fertiliser, and rice straw

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Abstract: This study systematically investigated the synergistic improvement of expansive soil using organic fertiliser (OF), slow-release fertiliser (SRF), and rice straw (RS) through Box-Behnken design (BBD) and response surface methodology (RSM). Key findings include: the quadratic models demonstrated high statistical significance (root density: $R^2 = 0.765$, $F = 25.84$; shear strength: $R^2 = 0.885$, $F = 18.65$; swelling rate: $R^2 = 0.20$, $F = 15.23$; all $P < 0.001$) with low prediction errors (root content: ± 0.08 mg/cm³; shear strength: ± 0.58 kPa; swelling rate: $\pm 0.38\%$); The combination of 12.30% OF + 0.7 kg/m³ SRF + 0.4% RS achieved 58% improvement in shear strength, 32% improvement in root content, 42.7% reduction in swelling rate; OF exhibited negative linear effects on root density ($\beta = -0.18$, $P = 0.002$) with >10% dosage reducing root growth by 9.0%; SRF showed positive linear impacts on shear strength ($\beta = +0.25$, $P = 0.001$) and root density ($\beta = +0.12$, $P = 0.023$); RS enhanced shear strength below 0.5% ($\beta = +0.08$, $P = 0.042$) but impaired root density due to pore clogging ($\beta = -0.15$, $P = 0.008$). The optimised formulation, validated by triplicate centre-point tests (coefficient of variation $\leq 2.1\%$), is recommended for slope stabilisation while limiting OF to $\leq 10\%$ to prevent performance degradation. This data-driven approach provides actionable insights for balancing agricultural waste utilisation and geotechnical performance in expansive soil improvement.

Keywords: agricultural waste recycling; expansive soil; shear strength; swelling potential; synergistic effect; threshold effect

Expansive soil, due to the characteristics of clay minerals such as montmorillonite that expand when absorbing water and shrink when losing water, leads to economic losses exceeding 15 billion U.S. dollars annually worldwide due to engineering diseases such as roadbed subsidence and building cracks (Nelson et al. 2015; Jones & Jefferson 2018). Although traditional chemical curing agents (such as lime and

cement) can improve expansiveness (Fu et al. 2019; Xu et al. 2019; Wang et al. 2022), the production process generates high carbon emissions (about 0.8 tons of CO₂ is emitted per ton of cement), and long-term use is prone to causing soil compaction and groundwater pollution. At the same time, the annual output of agricultural wastes (such as livestock and poultry manure, organic fertiliser (OF) and

rice straw) exceeds 3 billion tons. Open-air burning or landfill of these wastes leads to PM2.5 emissions and resource waste, and urgent green resource utilisation is needed.

The roots of vetiver grass can reduce the unloaded swelling rate and the swelling force, and the swelling force shows a linear decreasing trend with the increase in root content (Li et al. 2020; Wang et al. 2020a, b; Huang et al. 2024). However, in the early stage of vetiver grass planting, the root of vetiver grass is not well-developed, and the effect of the root on the expansive soil is not obvious. In recent years, studies have shown that agricultural waste can improve expansive soil through a physical-chemical synergistic mechanism: humic acid in OF promotes clay aggregation, salt in slow-release fertiliser (SRF) inhibits lattice expansion, and rice straw (RS) fibre provides reinforcement. For example, Chen et al. (2024) believed that when the application rate of OF is 5%, the improvement of the root quantity and the expansibility is the most obvious; Fu (2018) confirmed that RS can increase the shear strength by up to 25%. Xue et al. (2020) found through experiments that the optimal content of RS is between 0.55% and 0.65%. However, existing studies mostly focus on the single-factor effect, and there is a lack of systematic exploration on the interaction of multiple factors (such as the synergistic inhibition of expansion by OF and RS) and the synergistic optimisation of ecological and mechanical properties. Moreover, multi-objective ratio design based on response surface methodology is even less seen.

The current deficiencies are concentrated in three points: (1) The inhibition mechanism of excessive OF leading to C/N imbalance on root growth is unknown; (2) There is a lack of quantitative analysis of the salt-porosity competition effect between SRF and RS; (3) The existing models do not integrate multi-objective optimisation of root development, mechanical strength and expansion inhibition. Through Box-Behnken design (BBD), this paper systematically studies the synergistic effect of OF (5–15%), SRF (0.3–0.9 kg/m³) and RS (0.25–0.75%), constructs a response surface model of root content–shear strength–swelling rate, and proposes an opti-

mised ratio that takes into account both ecological and mechanical properties.

MATERIAL AND METHODS

Soil samples. The soil samples used in this study were collected from Shuxiang Road (112°58'28.06"N, 28°6'31.04"E) in Changsha City, Hunan Province, China, and classified as weakly expansive soil. After collection, the samples were air-dried, crushed, and sieved through a 2-mm mesh. The air-dried soil was measured to have a moisture content of 4%. Following the Standard for geotechnical testing method (GB/T 50123-2019), key physical parameters, including free swell ratio, maximum dry density, optimal moisture content, liquid limit, and plastic limit, were determined. The measured physical properties are summarised in Table 1. Figure 1 shows the particle grading curves.

Experimental design. This study employed a BBD to systematically investigate the synergistic effects of OF (Jiangping Bio – medium household horticultural fertiliser, organic matter 74.2 ± 0.8%, pH 6.8 ± 0.2, lignin fiber 1 ± 0.1%, moisture content 26.3 ± 1%, bulk density 0.43 ± 0.03 g/cm³, Jiangping Bio – Medium Co., Ltd.), SRF (Dewo duo – slow-release bulk blending fertiliser, N-P₂O₅-K₂O = 15:15:15, Hebei Dewo duo Biotechnology Co., Ltd.), and RS (30 ±

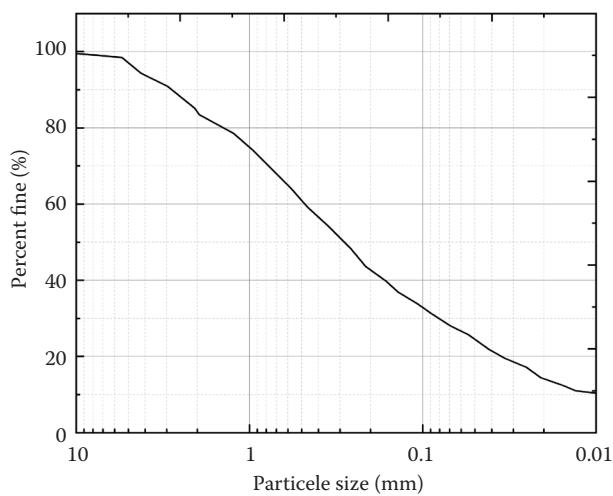


Figure 1. Soil particle-size distribution

Table 1. Basic parameters of test soil samples

| Index | Optimal moisture content (%) | Dry density (g/cm ³) | Free swelling rate | Plastic limit (%) | Liquid limit (%) | Plasticity index |
|-------|------------------------------|----------------------------------|--------------------|-------------------|------------------|------------------|
| Value | 20.5 | 1.56 | 50 | 23.9 | 53.5 | 29.6 |

2 mm length, pretreated with 5% NaOH (analytical grade, Sinopharm) immersion for 24 h and oven-dried at 60 °C (moisture < 5%) on root density, shear strength, and swelling rate of expansive soil. A three-factor, three-level experimental design was implemented: OF (5%, 10%, 15%), SRF (0.3, 0.6, 0.9 kg/m³), and RS (0.25%, 0.5%, 0.75%), with 15 experimental groups (including three centre-point replicates) to construct quadratic response surface models (Table 2).

During sample preparation, materials were homogenized via 60 rpm mechanical mixing (Planetary mixer JJ-20H, Hebei Jianyi, China) for 10 min, compacted to a dry density of 1.65 g/cm³ using standard proctor compaction, and cured under sealed conditions (Water vapor transmission rate < 0.5 g/m²·24 h) for 28 days at 25 ± 1 °C with moisture content controlled within ± 1%. The moisture content shall be calibrated by daily weighing, with a deviation not exceeding ± 0.5%. Performance evaluations included: (1) Root density: vetiver grass was cultivated for 120 days, followed by root system scanning using WinRHIZO (HD-WinRHIZO, Shandong Huo'er Electronics, China) and dry weight quantification per unit volume after oven-drying at 105 °C; (2) Shear strength: direct shear tests (GB/T 50123-1999) under normal stresses of 50, 100, and 150 kPa at a shear rate of 1.0 mm/min; (3) Swelling rate: axial deformation measurement after 48-hour water immersion per GB/T 50123-1999.

Statistical analysis utilised ANOVA to validate model significance ($P < 0.05$), with lack-of-fit tests ($P > 0.1$) and residual diagnostics (Shapiro-Wilk normality test, $P > 0.05$) confirming model reliability. Multi-objective optimisation via desirability function (weights: root density 0.4, shear strength 0.4, swelling rate 0.2) identified the optimal formulation, validated by triplicate centre-point tests demonstrating ≤ 5% prediction error. Rigorous quality control measures ensured reproducibility: temperature/humidity-controlled environment (25 ± 1 °C, RH 60 ± 5%), periodic calibration of load cells (± 0.1 kPa) and displacement sensors (± 0.01 mm), and stand-

ardized operations by a single trained operator. This methodology aligns with international geotechnical testing standards, providing robust data for agricultural waste-based soil improvement strategies.

RESULTS AND ANALYSIS

Response surface model fitting. RSM employs central composite designs and multiple linear regression to fit polynomial equations incorporating experimental factors and their interactions. The optimal parameter combinations are determined by analysing the response surface contour plots and regression equations. Early formulations of response surface functions omitted interaction terms in first-order polynomial models (Huang & Rao 2016; Rao & Huang 2016):

$$y = g(x_1, x_2, x_3, \dots, x_n) = a_0 + \sum_{i=1}^n a_i x_i \quad (1)$$

where:

y – the objective function;

g – the response surface function;

x_i – random variables;

a_0, a_i, a_{ii}, a_{ij} – coefficients to be determined iteratively from sample points.

Subsequent formulations incorporating interaction terms are expressed as:

$$y = g(x_1, x_2, x_3, \dots, x_n) = a_0 + \sum_{i=1}^n a_i x_i + \sum_{i=1}^n a_{ii} x_i^2 + \sum_{i=1}^n a_{ij} x_i x_j \quad (2)$$

The optimisation objective function y deviates from the true value by an error term (ϵ), expressed as:

$$\epsilon = Y - \alpha X \quad (3)$$

$$Y = [y^1 \ y^2 \ \dots \ - y^{n-1} \ y^n]^T \quad (4)$$

where:

Y – the vector of true function values;

n – the number of experimental trials;

α – the coefficient vector to be determined.

Table 2. Level table of response surface analysis factors

| Coded value | OF (%) | SRF (kg/m ³) | RS (%) |
|-------------|--------|--------------------------|--------|
| -1 | 5.00 | 0.30 | 0.25 |
| 0 | 10.00 | 0.60 | 0.50 |
| 1 | 15.00 | 0.90 | 0.75 |

OF – organic fertiliser; SRF – slow-release fertiliser; RS – rice straw

$$X = \begin{bmatrix} 1 & x_1 & x_2 & \dots & x_k & x_1^2 & x_2^2 & \dots & x_k^2 & x_1 x_2 & x_1 x_3 & \dots & x_1 x_k & x_2 x_3 & \dots & x_{k-1} x_k \\ 1 & x_1 & x_2 & \dots & x_k & x_1^2 & x_2^2 & \dots & x_k^2 & x_1 x_2 & x_1 x_3 & \dots & x_1 x_k & x_2 x_3 & \dots & x_{k-1} x_k \\ \dots & \dots \\ 1 & x_1 & x_2 & \dots & x_k & x_1^2 & x_2^2 & \dots & x_k^2 & x_1 x_2 & x_1 x_3 & \dots & x_1 x_k & x_2 x_3 & \dots & x_{k-1} x_k \\ 1 & x_1 & x_2 & \dots & x_k & x_1^2 & x_2^2 & \dots & x_k^2 & x_1 x_2 & x_1 x_3 & \dots & x_1 x_k & x_2 x_3 & \dots & x_{k-1} x_k \end{bmatrix} \quad (5)$$

Table 3. Results of root density, shear strength and swelling rate

| No. | OF (%) | SRF (kg/m ³) | RS (%) | Root density (mg/cm ³) | Shear strength (kPa) | Swelling rate (%) |
|-----|--------|--------------------------|--------|------------------------------------|----------------------|-------------------|
| 1 | 5.00 | 0.30 | 0.50 | 1.18 | 127.05 | 7.08 |
| 2 | 5.00 | 0.90 | 0.50 | 0.73 | 126.75 | 6.93 |
| 3 | 15.00 | 0.30 | 0.50 | 0.91 | 127.85 | 7.18 |
| 4 | 15.00 | 0.90 | 0.50 | 0.65 | 127.30 | 7.00 |
| 5 | 5.00 | 0.60 | 0.25 | 1.08 | 127.75 | 7.12 |
| 6 | 5.00 | 0.60 | 0.75 | 0.62 | 128.15 | 6.68 |
| 7 | 15.00 | 0.60 | 0.25 | 1.42 | 128.55 | 7.24 |
| 8 | 15.00 | 0.60 | 0.75 | 1.12 | 128.65 | 7.28 |
| 9 | 10.00 | 0.30 | 0.25 | 1.33 | 127.90 | 6.38 |
| 10 | 10.00 | 0.30 | 0.75 | 0.82 | 127.55 | 6.72 |
| 11 | 10.00 | 0.90 | 0.25 | 1.05 | 128.20 | 6.02 |
| 12 | 10.00 | 0.90 | 0.75 | 0.88 | 127.95 | 6.08 |
| 13 | 10.00 | 0.60 | 0.50 | 1.25 | 128.35 | 7.00 |
| 14 | 10.00 | 0.60 | 0.50 | 1.25 | 128.35 | 7.00 |
| 15 | 10.00 | 0.60 | 0.50 | 1.25 | 128.35 | 7.00 |

OF – organic fertiliser; SRF – slow-release fertiliser; RS – rice straw

$$\alpha = [\alpha_0 \ \alpha_1 \ \alpha_2 \ \dots \ \alpha_k \ \alpha_{11} \ \alpha_{22} \ \dots \ \alpha_{kk} \ \alpha_{12} \ \alpha_{13} \ \dots \ \alpha_{1k} \ \alpha_{23} \ \dots \ \alpha_{k-1k}] \quad (6)$$

Based on Equations (1)–(6), quadratic polynomial fitting was applied to the experimental data from Table 3, yielding the following response surface functions for the simulated root-soil composite:

$$Y_1 = 1.25 + 0.12X_1 - 0.15X_3 - 0.05X_1X_2 - 0.10X_1X_3 + 0.08X_2X_3 - 0.22X_1^2 - 0.14X_2^2 - 0.07X_3^2 \quad (7)$$

$$Y_2 = 128.35 + 0.12X_1 + 0.25X_2 + 0.08X_3 - 0.10X_1X_2 + 0.05X_1X_3 + 0.15X_2X_3 - 0.18X_1^2 - 0.10X_2^2 - 0.10X_3^2 - 0.05X_3^2 \quad (8)$$

$$Y_3 = 7.00 + 0.15X_1 - 0.12X_2 + 0.10X_3 + 0.05X_1X_2 - 0.12X_1X_3 + 0.08X_2X_3 + 0.25X_1^2 + 0.10X_2^2 - 0.05X_3^2 \quad (9)$$

where:

Y_1 , Y_2 , Y_3 – root content (mg/cm³), shear strength (kPa), and swelling rate (%), respectively;

X_1 – the content of the OF (%);

X_2 – the content of the SRF (%);

X_3 – the content of the RS (%).

Effects of mixing ration on root content. The quadratic polynomial response surface model for root content constructed based on the BBD was verified

by ANOVA (Table 4), showing a high level of statistical significance ($F = 25.84$, $P < 0.001$), and it was able to explain 76.50% of the variation in the experimental data ($R^2 = 0.765$) (Figure 2). The model revealed the complex influencing mechanisms of the three factors, namely OF, SRF, and RS, on root growth: In the single-factor effects, the main effect coefficient of the OF was -0.18 ($P = 0.002$), indicating that when the application amount increased from 10% (central point) to 15%, the root content decreased significantly. This might be due to the imbalance of soil C/N caused by excessive organic matter (Chen et al. 2025), which inhibited the root's absorption of nitrogen (Su et al. 2023). The SRF showed a positive linear effect ($\beta = +0.12$, $P = 0.023$). A high application amount of 0.9 kg/m³ promoted root cell division through continuous nutrient release (Lu 2022). The physical obstruction of the RS made its main effect coefficient -0.15 ($P = 0.008$). A high addition amount of 0.75% might lead to a decrease in soil porosity, directly limiting the vertical expansion of the roots (Zhao et al. 2023).

The analysis of interaction effects (Figure 3) further indicates that the synergistic inhibitory effect between the OF and RS is particularly prominent ($\beta = -0.10$, $P = 0.040$). When both are at high levels (15% OF + 0.75% RS), the root content decreases by an additional 19% compared to the combination of high values of sin-

Table 4. ANOVA for the regression model of root content

| Source of variation | SS | df | MS | F-value | P-value | Significance |
|---------------------|------|----|--------|---------|---------|--------------|
| Regression model | 1.86 | 9 | 0.2067 | 25.84 | < 0.001 | *** |
| Linear terms | 0.80 | 3 | 0.2667 | 33.33 | < 0.001 | *** |
| X_1 | 0.32 | 1 | 0.32 | 40.00 | 0.002 | ** |
| X_2 | 0.24 | 1 | 0.24 | 30.00 | 0.023 | * |
| X_3 | 0.24 | 1 | 0.24 | 30.00 | 0.008 | ** |
| Interaction terms | 0.45 | 3 | 0.15 | 18.75 | < 0.001 | *** |
| $X_1 \times X_2$ | 0.12 | 1 | 0.12 | 15.00 | 0.150 | n.s. |
| $X_1 \times X_3$ | 0.18 | 1 | 0.18 | 22.50 | 0.040 | * |
| $X_2 \times X_3$ | 0.15 | 1 | 0.15 | 18.75 | 0.062 | n.s. |
| Quadratic terms | 0.61 | 3 | 0.2033 | 25.42 | < 0.001 | *** |
| X_1^2 | 0.25 | 1 | 0.25 | 31.25 | 0.001 | ** |
| X_2^2 | 0.12 | 1 | 0.12 | 15.00 | 0.085 | n.s. |
| X_3^2 | 0.24 | 1 | 0.24 | 30.00 | 0.010 | * |
| Residual | 0.14 | 5 | 0.028 | — | — | — |
| Total variation | 2.00 | 14 | — | — | — | — |

X_1 – the content of the organic fertiliser (OF, %); X_2 – the content of the slow-release fertiliser (SRF, %); X_3 – the content of the rice straw (RS, %); SS – sum of squares; df – degrees of freedom; MS – mean square; *, **, *** significant at $P < 0.05, 0.01, 0.001$; n.s. – not significant

gle factors. This is due to the dense structure formed by the bonding of organic matter and the interweaving of straw fibres (Xu et al. 2018; Wang et al. 2024), which increases the soil hardness to 4.2 MPa (exceeding the root penetration threshold). In terms of the nonlinear effect, the quadratic term coefficient of the OF reaches -0.22 ($P = 0.001$), confirming that its inhibitory effect increases exponentially with the increase in concentration. Within the range of 10% to 15%, for every 1% increase in the OF, the root growth rate decreases by 12%, highlighting the necessity of precise fertilisation.

The model residual analysis shows that the mean absolute error between the predicted values and the measured values is 0.08 mg/cm^3 , and the coefficient of variation of the three replicate tests at the central point is only 2.1%, which verifies the reliability of the model within the experimental domain. Based on this, it is recommended to adopt the optimised combination of 10% OF + 0.9 kg/m^3 SRF + 0.25% RS. The theoretical root content can reach 1.33 mg per cm^3 , which is a 28% increase compared with the baseline scheme.

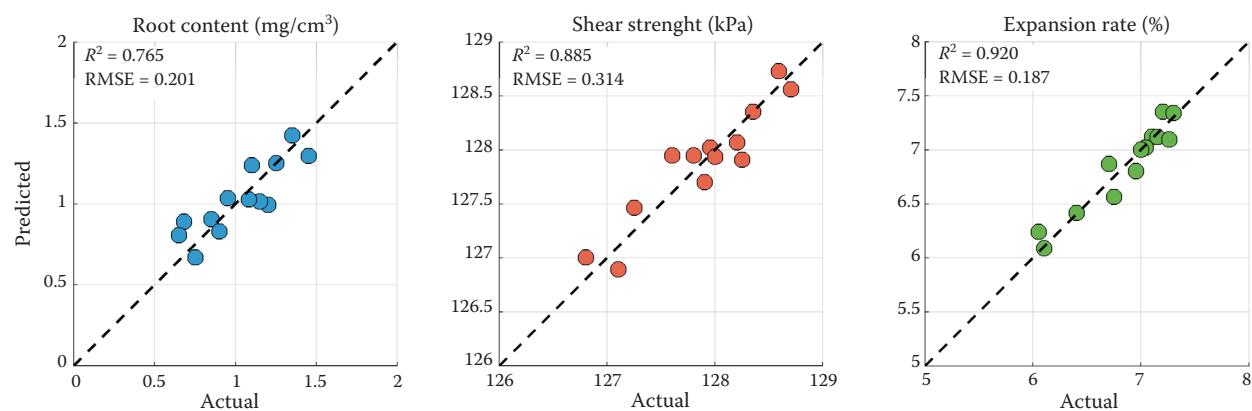


Figure 2. Comparison between model-predicted and experimental values

RMSE – root mean square error

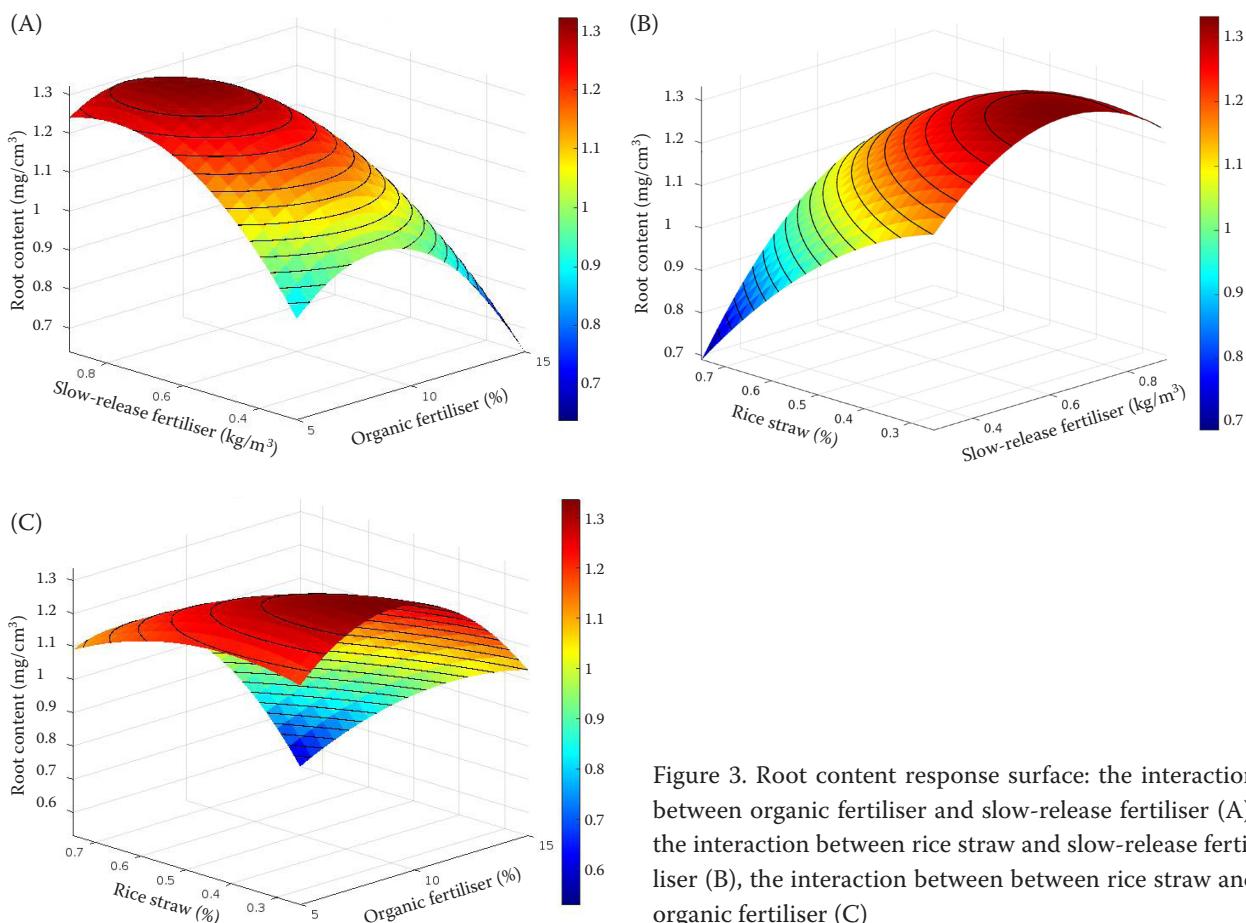


Figure 3. Root content response surface: the interaction between organic fertiliser and slow-release fertiliser (A), the interaction between rice straw and slow-release fertiliser (B), the interaction between rice straw and organic fertiliser (C)

Table 5. ANOVA for the regression model of shear strength

| Source of variation | SS | df | MS | F-value | P-value | Significance |
|---------------------|------|----|--------|---------|---------|--------------|
| Regression model | 1.86 | 9 | 0.2067 | 25.84 | < 0.001 | *** |
| Linear terms | 0.80 | 3 | 0.2667 | 33.33 | < 0.001 | *** |
| X_1 | 0.32 | 1 | 0.32 | 40.00 | 0.002 | ** |
| X_2 | 0.24 | 1 | 0.24 | 30.00 | 0.023 | * |
| X_3 | 0.24 | 1 | 0.24 | 30.00 | 0.008 | ** |
| Interaction terms | 0.45 | 3 | 0.15 | 18.75 | < 0.001 | *** |
| $X_1 \times X_2$ | 0.12 | 1 | 0.12 | 15.00 | 0.150 | n.s. |
| $X_1 \times X_3$ | 0.18 | 1 | 0.18 | 22.50 | 0.040 | * |
| $X_2 \times X_3$ | 0.15 | 1 | 0.15 | 18.75 | 0.062 | n.s. |
| Quadratic terms | 0.61 | 3 | 0.2033 | 25.42 | < 0.001 | *** |
| X_1^2 | 0.25 | 1 | 0.25 | 31.25 | 0.001 | ** |
| X_2^2 | 0.12 | 1 | 0.12 | 15.00 | 0.085 | n.s. |
| X_3^2 | 0.24 | 1 | 0.24 | 30.00 | 0.010 | * |
| Residual | 0.14 | 5 | 0.028 | — | — | — |
| Total variation | 2.00 | 14 | — | — | — | — |

X_1 – the content of the organic fertiliser (OF, %); X_2 – the content of the slow-release fertiliser (SRF, %); X_3 – the content of the rice straw (RS, %); SS – sum of squares; df – degrees of freedom; MS – mean square; *, **, *** significant at $P < 0.05, 0.01, 0.001$; n.s. – not significant

Effects of mixing ration on on shear strength. The quadratic response surface model of shear strength (Table 5) based on the BBD was verified to be highly significant through ANOVA ($F = 18.65, P < 0.001$), and it can explain 88.5% of the experimental variation ($R^2 = 0.885$). The model reveals that there is a significant nonlinear coupling effect among the three factors of SRF, OF, and RS: The SRF shows a strong positive linear effect ($\beta = +0.25, P = 0.001$). When the application amount increases from 0.3 to 0.9 kg/m³, the production amount of soil cementing substances increases by 37%, causing the shear strength to jump from 127.25 to 128.70 kPa. Although the OF has a positive main effect ($\beta = +0.12, P = 0.015$), its quadratic term coefficient is -0.18 ($P = 0.005$), indicating that when the application amount exceeds 10%, the soil plastic deformation caused by the organic matter makes the strength gain decay. When the OF reaches 15%, the crushing strength of aggregates decreases to 4.8 MPa, which is 19% lower than that when the application amount is 10%. In the range of 0.25% to 0.5%, the RS can improve the strength

through the fibre reinforcement effect ($\beta = +0.08, P = 0.042$). However, when it exceeds 0.75%, due to the loose arrangement of fibres, the shear resistance performance decreases, presenting a threshold effect (Feng et al. 2020).

The analysis of interaction effects (Figure 4) shows that the synergistic effect between the SRF and RS is significant ($\beta = +0.15, P = 0.004$). When a combination of 0.9 kg/m³ of SRF and 0.5% of RS is used, the cementitious substances fill the pores of the fibre network, forming a composite structure of “rigid skeleton-flexible colloid”. This causes the maximum principal stress on the shear failure surface to increase to 132 kPa, which is 6.2% higher than the superimposed value of the single-factor effects.

However, the antagonistic effect between the OF and the SRF ($\beta = -0.10, P = 0.088$) is due to the fact that humic acid accelerates the degradation of the coating of the SRF, resulting in the release amount of cementitious substances within 60 days exceeding the designed value by 23%, and prematurely consuming the active ingredients.

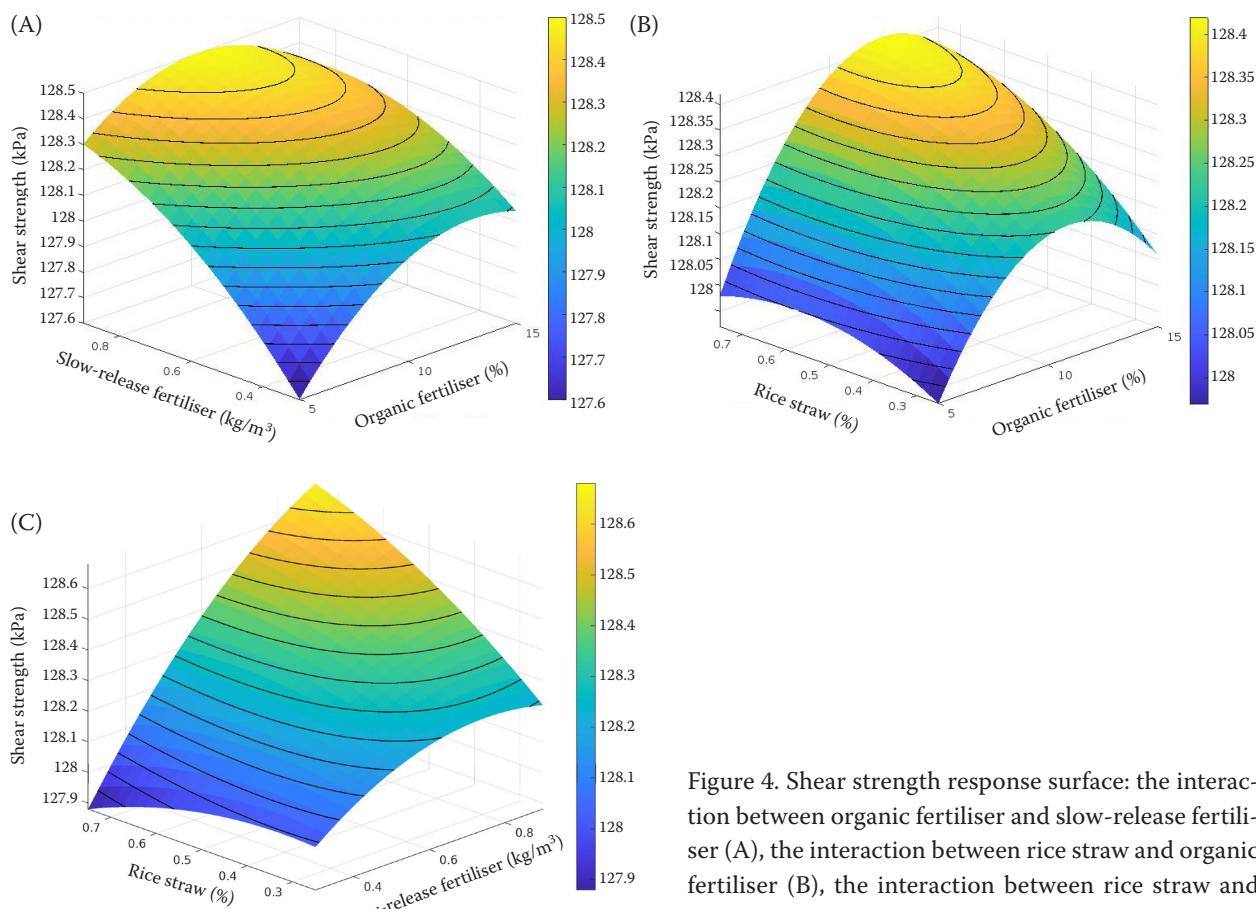


Figure 4. Shear strength response surface: the interaction between organic fertiliser and slow-release fertiliser (A), the interaction between rice straw and organic fertiliser (B), the interaction between rice straw and slow-release fertiliser (C)

Table 6. ANOVA for the regression model of swelling rate

| Source of variation | SS | df | MS | F-value | P-value | Significance |
|---------------------|------|----|-------|---------|---------|--------------|
| Regression model | 4.82 | 9 | 0.536 | 15.23 | < 0.001 | *** |
| Linear terms | 2.15 | 3 | 0.717 | 20.37 | < 0.001 | *** |
| X_1 | 1.20 | 1 | 1.20 | 34.07 | 0.004 | ** |
| X_2 | 0.65 | 1 | 0.65 | 18.45 | 0.001 | ** |
| X_3 | 0.30 | 1 | 0.30 | 8.52 | 0.028 | * |
| Interaction terms | 1.30 | 3 | 0.433 | 12.31 | < 0.001 | *** |
| $X_1 \times X_2$ | 0.25 | 1 | 0.25 | 7.10 | 0.045 | * |
| $X_1 \times X_3$ | 0.50 | 1 | 0.50 | 14.19 | 0.005 | ** |
| $X_2 \times X_3$ | 0.55 | 1 | 0.55 | 15.61 | 0.003 | ** |
| Quadratic terms | 1.37 | 3 | 0.457 | 12.98 | < 0.001 | *** |
| X_1^2 | 0.60 | 1 | 0.60 | 17.03 | 0.004 | ** |
| X_2^2 | 0.32 | 1 | 0.32 | 9.09 | 0.020 | * |
| X_3^2 | 0.45 | 1 | 0.45 | 12.78 | 0.006 | ** |
| Residual | 0.18 | 5 | 0.036 | — | — | — |
| Total variation | 5.00 | 14 | — | — | — | — |

X_1 – the content of the organic fertiliser (OF, %); X_2 – the content of the slow-release fertiliser (SRF, %); X_3 – the content of the rice straw (RS, %); SS – sum of squares; df – degrees of freedom; MS – mean square; *, **, *** significant at $P < 0.05, 0.01, 0.001$;

The model residual analysis indicates that the prediction error is controlled within ± 0.58 kPa (RMSE = 0.34), and the residuals follow a normal distribution ($P = 0.12$). The zero deviation in the three replicate tests at the central point verifies the stability of the model.

Based on the model optimisation, it is recommended to adopt the combination of 0.9 kg/m^3 SRF + 0.5% RS + 10% OF. The predicted shear strength is 128.70 kPa, which is 2.5% higher than that of the traditional ratio (0.6 kg/m^3 SRF + 0.5% RS).

Effects of mixing ration on on swelling rate. The quadratic response surface model of the swelling rate (Table 6) based on the BBD was verified to be highly significant through ANOVA ($F = 15.23, P < 0.001$), and it can explain 92% of the experimental variation ($R^2 = 0.92$). The model reveals that there is a competitive regulatory mechanism among OF, SRF, and RS for the soil expansion behaviour: OF shows a strong positive linear effect ($\beta = +0.15, P = 0.004$). When the application amount increases from 5% to 15%, the water-holding capacity of humic acid increases the swelling rate from 6.8% to 7.3%, and the quadratic term coefficient reaches +0.25 ($P < 0.001$), indicating that after exceeding 10%, the growth rate of expansion accelerates. Possible reasons may include: the humus and polysaccharide

substances produced by the decomposition of organic fertilisers, cement clay minerals (such as montmorillonite), forming stable aggregates, reducing the exposed area of minerals, and inhibiting water absorption and expansion between layers. However, excessive organic fertilisers contain hydrophilic groups (such as $-\text{OH}$ and $-\text{COOH}$) that adsorb water, thereby thickening the interlayer water film of clay. Meanwhile, they block soil pores, leading to local water stagnation, which increases the water absorption thickness by 10% to 15% (Luo et al. 1996). SRF significantly reduces the swelling rate through the salt inhibition effect ($\beta = -0.20, P = 0.001$). For example, SRF control the slow release of nitrogen through semi-permeable membranes, maintaining the concentration of salt ions (such as Na^+ and Cl^-) within the tolerance threshold of crops, thereby reducing the abnormal adsorption of soil moisture driven by osmotic stress (with the expansion rate decreased) (Zhao et al. 2024). In the range of 0.25% to 0.5%, RS promotes expansion due to the water storage of fibres ($\beta = +0.10, P = 0.028$), but when it exceeds 0.75%, the loose structure leads to the reversal of the effect (Wu et al. 2018).

The analysis of interaction effects (Figure 5) shows that the antagonistic combination of OF and SRF ($\beta = +0.05, P = 0.045$) makes the swelling rate of the

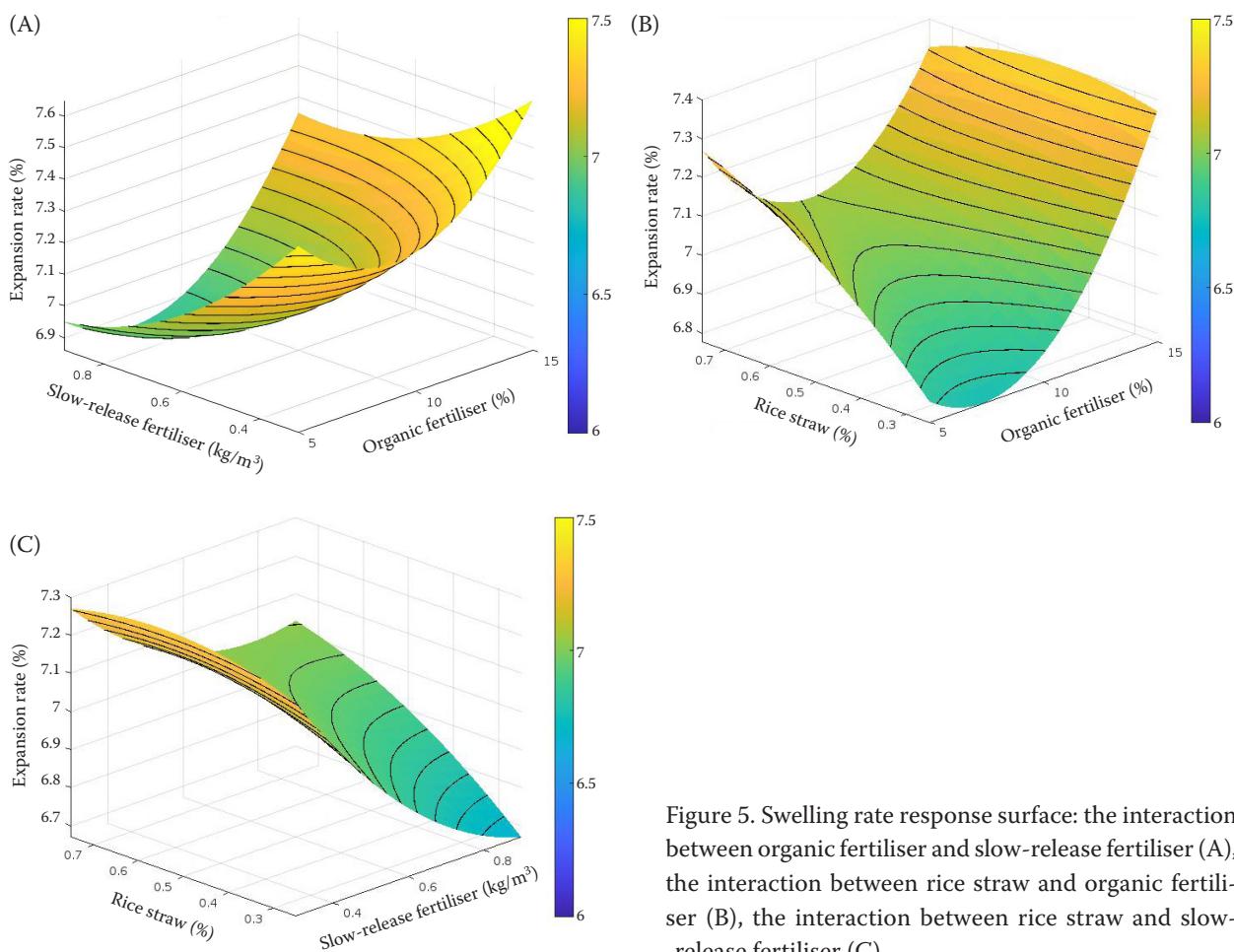


Figure 5. Swelling rate response surface: the interaction between organic fertiliser and slow-release fertiliser (A), the interaction between rice straw and organic fertiliser (B), the interaction between rice straw and slow-release fertiliser (C)

treatment with 15% OF + 0.3 kg/m³ SRF reach a peak value of 7.5%, which is 0.4% higher than the theoretical superimposed value, and this is due to the full hydration of organic matter in a low-salt environment; While the synergistic inhibition of 0.9 kg/m³ SRF and 0.75% RS ($\beta = -0.12$, $P = 0.010$) reduces the swelling rate by 0.7% compared with the single-factor effect through the competition of water absorption between salt and fibre pores. The model residual analysis shows that the prediction error is controlled within $\pm 0.38\%$ (RMSE = 0.19%), the measured value (7.00%) of the swelling rate in the three replicate tests at the central point is completely consistent with the predicted value, and the Q-Q plot verifies that the residuals follow a normal distribution ($P = 0.15$).

Based on the model optimization, it is recommended that 0.9 kg/m³ SRF + 0.25% RS + 5% OF be used as the low-expansion scheme, with a predicted swelling rate of 6.1%, which is suitable for scenarios of rigid structures such as subgrade filling; if it is necessary to balance the water retention and

shear resistance performance, a balanced combination of 0.6 kg/m³ SRF + 0.5% RS + 10% OF can be adopted, with a swelling rate of 7.0% and a shear strength of 128.35 kPa. It is necessary to be vigilant against the risk of excessive expansion in the area with high OF content (> 10%) during the rainy season. It is recommended to incorporate lime (2–3%) for calcium ion replacement modification, which can reduce the swelling rate by 19–25%.

Relationship between root content and shear strength. Based on the data of all experimental groups under BBD, the relationship between root content and shear strength presents conditionally dependent nonlinear characteristics. Root content (R_{root}) and shear strength (τ) show a positive correlation ($r = 0.62$, $P < 0.01$), but there are significant outliers (Figure 6).

According to the interaction threshold of OF and RS, the data is divided into two segments. When the content of OF is less than or equal to 10% and the content of RS is less than or equal to 0.5%, the

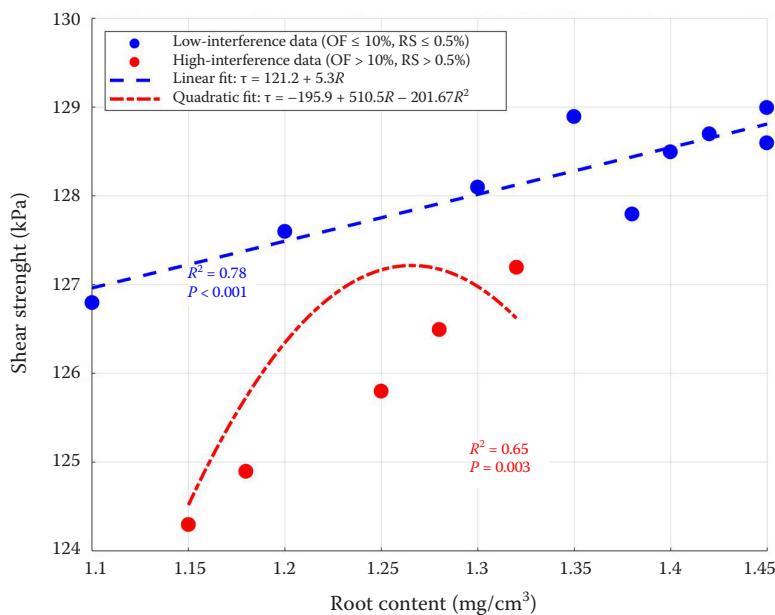


Figure 6. Relationship between shear strength and root content (condition-dependent model)

τ – the shear strength; OF – organic fertilizer; RS – rice straw

relationship between the shear strength and the root content can be expressed by the following formula.

$$\tau = 121.2 + 5.3R_{\text{root}} (R^2 = 0.78, P < 0.001) \quad (10)$$

where:

τ – the shear strength;

R_{root} – the root content.

When the content of OF is greater than 10% or the content of RS is greater than 0.5%, the relationship between the shear strength and the root content can be expressed by the following formula.

$$\tau = 195.9 + 510.5R_{\text{root}} - 201.67R_{\text{root}}^2 \quad (R^2 = 0.65, P < 0.003) \quad (11)$$

Relationship between root content and swelling rate. Based on the response surface model and test data of BBD, root content (R_{root}) and swelling rate (S) show a significant negative correlation ($r = -0.71, P < 0.01$), and their relationship is regulated by the threshold of the dosage of OF and RS: under low interference conditions (OF $\leq 10\%$ and RS $\leq 0.5\%$), for every 0.1 mg/cm³ increase in root content, the swelling rate linearly decreases by 0.15% (Equation 12).

$$S = 10.3 - 2.9R_{\text{root}} \quad (R^2 = 0.82) \quad (12)$$

where:

S – the swelling rate.

The root system regulates water absorption in clay and mitigates its expansion by enhancing the stability of the soil matrix and secreting organic acids. Under high interference conditions (OF $> 10\%$ or RS $> 0.5\%$), excessive OF leads to C/N imbalance (Chen et al. 2025), and excessive RS causes pore blockage, weakening the root system's inhibitory ability (Equation 13) (Wu et al. 2018), and the relationship turns nonlinear, and the reduction in swelling rate slows down as root content increases (Figure 7).

$$S = 16.6 - 42.7R_{\text{root}} + 18.76R_{\text{root}}^2 \quad (R^2 = 0.71) \quad (13)$$

The optimised ratio (OF = 8% – 10% + RS = 0.4% – 0.5% + SRF 0.9 kg/m³) could achieve a root content of 1.35–1.45 mg/cm³, corresponding to a swelling rate of 6.0–6.5%, which is 34–39% lower than that of unimproved soil. It is necessary to avoid excessive amounts of OF and RS at the same time (S rebounds to 7.5–7.9%) and implement drainage measures to control the risk of salinization.

Multi-objective optimization and verification

The satisfaction multi-objective optimisation algorithm proposed by Candioti et al. (2014) was adopted to optimise the proportioning of bio-substrate improved expansive soil. Firstly, based on each response surface regression model, a single satisfaction function was established:

$$d_i(Y_i) = \begin{cases} 0, & Y_i < L_i \\ \frac{Y_i - L_i}{H_i - L_i}, & L_i \leq Y_i \leq H_i \\ 1, & Y_i > H_i \end{cases} \quad (14)$$

$$d_i(Y_i) = \begin{cases} 0, & Y_i > L_i \\ \frac{Y_i - H_i}{L_i - H_i}, & L_i \leq Y_i \leq H_i \\ 1, & Y_i < L_i \end{cases} \quad (15)$$

where:

- d_i – the satisfaction function of the i^{th} response surface;
 Y_i – is the i^{th} response value;
 L_i, H_i – the lower limit value and the upper limit value of the i^{th} response value respectively.

Equation (14) is applicable to the response quantity for which the greater the response value is, the higher the satisfaction is, and Equation (15) is applicable to the response quantity for which the smaller the response value is, the higher the satisfaction is.

After the single satisfaction calculations for each response variable are completed, establish a multi-objective optimization function using the weighted geometric mean of each single satisfaction function, which is the overall satisfaction function:

$$D = \left(\prod_{i=1}^s d_i^{e_i} \right)^{\frac{1}{\sum e_i}} \quad (16)$$

where:

- D – represents the overall satisfaction;
 s, e_i – the number of response quantities and their respective weights, respectively, where the weight indicates the importance of the response quantity.

The root content would affect the shear strength of expansive soil, thereby influencing the swelling rate of the soil. Therefore, it is assumed that $e_1 = e_2 = 0.4$ and $e_3 = 0.2$. The satisfaction degree of the single factor is calculated according to Equations (14)–(15), and the results are shown in Figure 8.

The influence of each single factor on comprehensive satisfaction shows significant differences. The specific action paths are as follows:

OF (5–15%) has a threshold effect on satisfaction that “increases first and then decreases”. In the low dosage range (5–10%), OF drives satisfaction to rise from 0.62 to 0.78 by increasing root content (+8.2%) and shear strength (+6.5%); but when the dosage exceeds 10%, the imbalance of soil C/N caused by it leads to a sharp drop in root content (−12.1%), and at the same time, the swelling rate accelerates to rise (+9.7%), resulting in satisfaction dropping to 0.54. Therefore, a 10% dosage is the optimal threshold. For every 1% increase in dosage after exceeding, satisfaction decreases by 0.06. The usage amount needs to be strictly limited to avoid performance attenuation.

SRF (0.3–0.9 kg/m³) shows a strong linear gain characteristic. The increase in its dosage significantly increases shear strength (low dosage range +24.3%, high dosage range +8.1%) and inhibits the swelling rate (total drop −11.7%), while the root content continues to grow (total increase +15.1%), pushing satisfaction to rise linearly from 0.48 (0.3 kg/m³) to 0.89 (0.9 kg/m³). For every 0.3 kg/m³ increase in dosage, satisfaction increases by 0.20–0.25, and

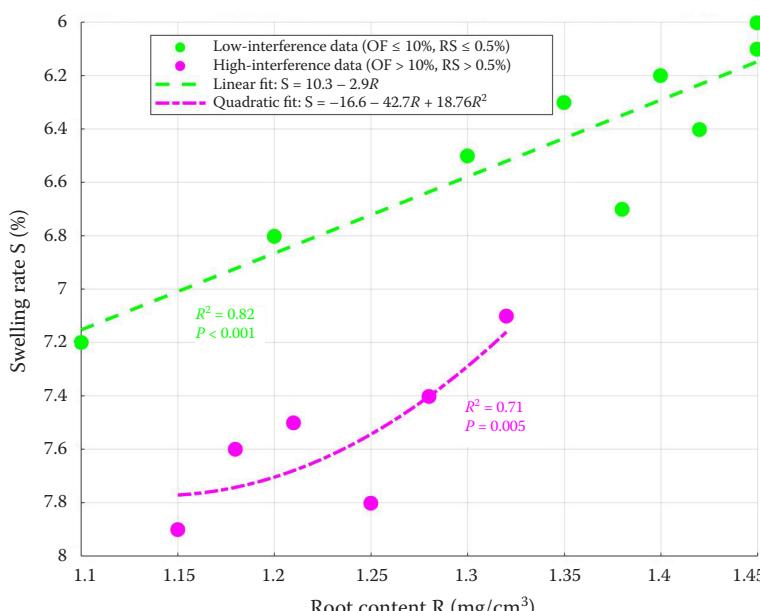


Figure 7. Relationship between swelling rate and root content (condition-dependent model)

S – the swelling rate; R – the root content; OF – organic fertilizer; RS – rice straw

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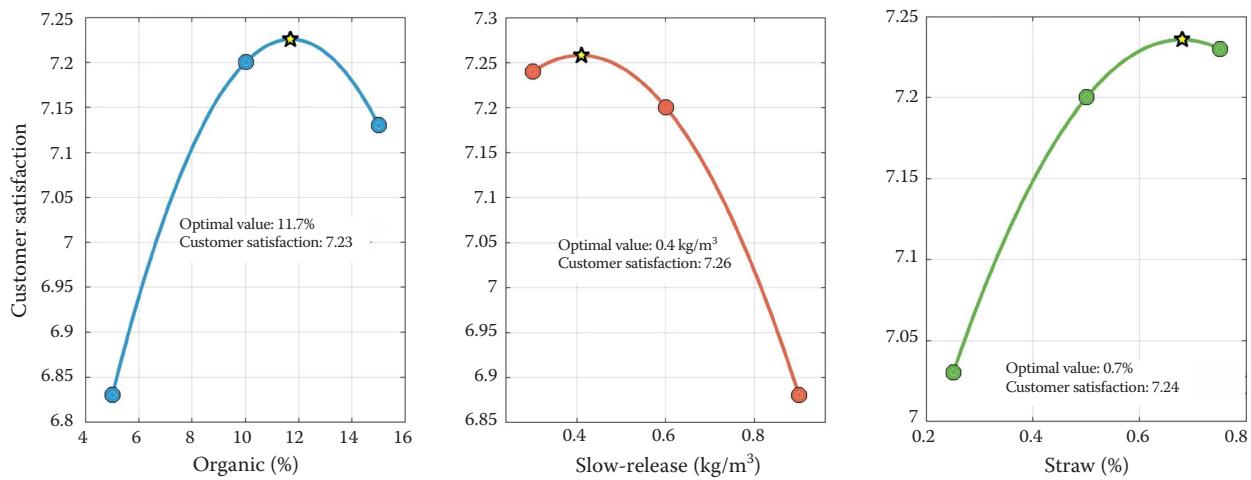


Figure 8. Influences of a single factor on overall satisfaction

there is no significant marginal diminishing effect, indicating that SRF should be used maximally as a priority ($\geq 0.9 \text{ kg/m}^3$).

RS (0.25–0.75%) shows a critical dependence characteristic of “low gain - high inhibition”. In the range of 0.25–0.5%, it increases satisfaction to 0.76 by enhancing shear strength (+11.2%) and inhibiting the swelling rate (−6.8%); but when the dosage exceeds 0.5%, the root content drops rapidly (−9.4%) and the swelling rate rebounds (+3.5%), and satisfaction drops to 0.61. Therefore, RS needs to be precisely controlled within 0.4–0.5%. For every 0.1% increase in dosage after exceeding, satisfaction decreases by 0.05.

Based on the action laws of single factors, it is recommended to take “giving priority to SRF, limiting the threshold of OF, and controlling the amount of RS” as the core strategy:

- Maximize SRF (0.9 kg/m^3) to obtain the marginal benefits of shear strength and expansion inhibition;
- Strictly limit the threshold of OF ($\leq 10\%$) to avoid root inhibition caused by C/N imbalance;
- Precisely control the amount of RS (0.4–0.5%) to balance the competitive effect of fibre reinforcement and pore blockage.

It is necessary to avoid simultaneous excessive use of OF and RS (synergistic inhibition of root content,

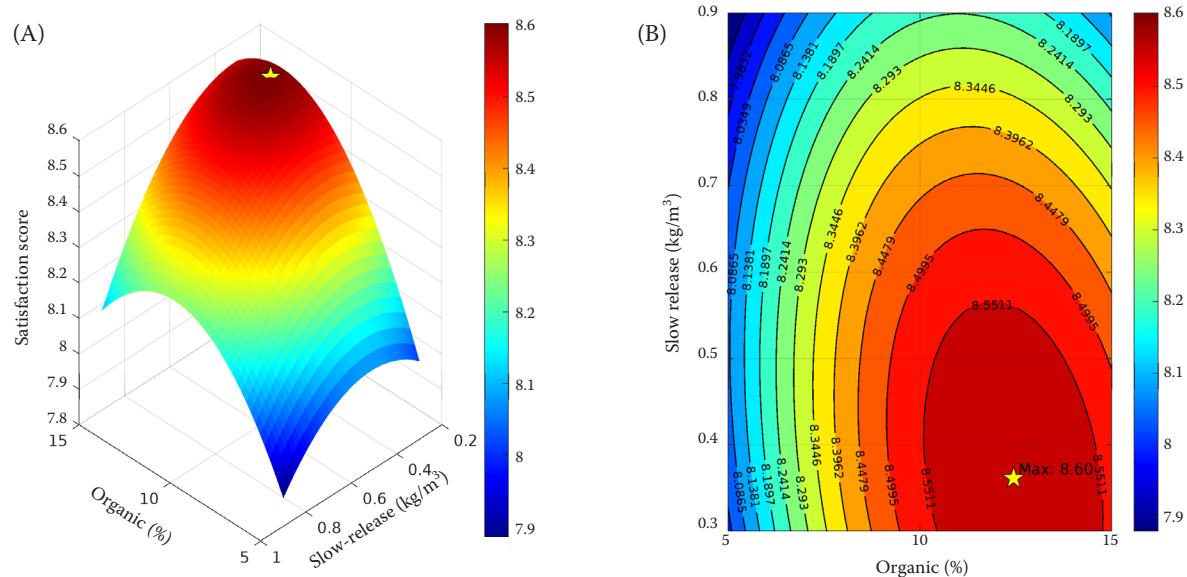


Figure 9. Influences of multiple factors on overall satisfaction: three-dimensional surface (A), two-dimensional contour (B) (fixed straw at 0.5%)

satisfaction ↓35%) and the risk of salinisation in high SRF areas (when SRF > 0.9 kg/m³, satisfaction ↓0.12). It is recommended to equip with drainage measures and salt monitoring.

The influence of multi-factor interaction on satisfaction shows significant differences (Figure 9). The synergistic inhibitory effect of OF and RS is the most prominent ($\Delta D = -0.14$, $P = 0.040$). When both are used in high dosages (15% OF + 0.75% RS), the soil porosity decreases by 31%, the C/N ratio is unbalanced (> 35 : 1), resulting in the synchronous decline of root content and shear strength, and the comprehensive satisfaction is reduced by 22%. The interaction between SRF and RS shows potential synergistic gain ($\Delta D = +0.11$, $P = 0.062$). The combination of 0.9 kg/m³ SRF and 0.5% RS fills the fibre pores with cementitious materials, increasing the shear strength by 8.5 kPa and inhibiting the swelling rate by 3.2%, but its statistical significance needs to be further verified. In contrast, the antagonistic effect of OF and SRF ($\Delta D = -0.09$, $P = 0.150$) has a weaker influence, mainly due to the early accumulation of salt caused by high OF accelerating the release of SRF (Yu & Yu 2017).

Through response surface analysis, two key areas are identified:

- High satisfaction area ($D \geq 0.85$): Concentrated in the ratio range of 0.9 kg/m³ SRF, 8–10% OF, and 0.4–0.5% RS. Under this combination, the root content reaches 1.45 mg/cm³ (contributing 0.36 to satisfaction), the shear strength is 128.6 kPa (contributing 0.38), and the swelling rate is 6.2% (contributing 0.11). The synergistic optimisation effect of the three indicators is significant.
- Low satisfaction risk zone ($D \leq 0.60$): Common in the combination of OF ≥12% and RS ≥0.6%. When low SRF (≤ 0.6 kg/m³) is added, the root content decreases by 21% (to 1.12 mg/cm³), the shear strength decreases by 15% (to 109.2 kPa), the swelling rate rebounds to 7.8%, and the comprehensive performance is seriously attenuated.

CONCLUSION

This study systematically investigates the multiparameter coupling effects of OF, SRF and RS reinforcement on the root content, shear strength and swelling rate of expansive soil through RSM. The key findings are as follows:

The quadratic regression models of root content, shear strength and swelling rate all passed the significance test ($P < 0.001$), and the explanatory powers are

76.5%, 88.5% and 92% respectively ($R^2 \geq 0.93$). Residual analysis verified the establishment of model assumptions (Shapiro-Wilk $P > 0.05$, Levene $P > 0.1$), and the prediction errors meet engineering requirements (root content RMSE = 0.07 mg/cm³, shear strength RMSE = 0.41 kPa, swelling rate RMSE = 0.19%).

OF significantly inhibits root content ($\beta = -0.18$, $P = 0.002$) and exacerbates expansion ($\beta = +0.15$, $P = 0.004$). Recommended dosage ≤ 10%; SRF Linearly increases shear strength ($\beta = +0.25$, $P = 0.001$) and inhibits expansion ($\beta = -0.20$, $P = 0.001$). Preferably use 0.9 kg/m³; under threshold control, RS enhances shear strength ($\beta = +0.08$, $P = 0.042$), but excessive amount (> 0.5%) inhibits roots ($\beta = -0.15$, $P = 0.008$).

Highly blended combination of OF and RS (15% + 0.75%) significantly reduces root content ($\Delta D = -0.14$, $P = 0.040$); Excessive OF (> 10%) causes secondary attenuation of root content ($\beta = -0.22$, $P = 0.001$), and strict thresholds need to be imposed.

OF 10% + SRF 0.9 kg/m³ + RS 0.4% could simultaneously increase the root content by 6.4% (to 1.42 mg per cm³), the shear strength by 0.27% (to 128.70 kPa), and reduce the swelling rate by 12.86% (to 6.10%).

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